KANTON LUZERN
Bildungs- und Kulturdepartement

Kantonsschule Alpenquai Luzern

Written Matura Exam 2018

Subject	Mathematics Basic Course
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Classes	6l / 6k
Date of the exam	May 25, 2018
Time	180 minutes
Aids allowed	- "Mathematics Formulary", Adrian Wetzel - A dictionary (book, no electronic translator) - TI-30, Voyage 200 without user manual
Instructions	- Importance is attached to a proper and clear representation. - Write each exercise on a separate sheet of paper. - All solutions must show the steps leading to the result. - Put your personal number, your name and your class on every sheet of paper.
Maximum points per exercise	Exercise 1: 12.5 Exercise 2: 11 Exercise 3: 12 Exercise 4: 12.5 **Total: 48** 43 points are required for a grade of 6.
Number of pages	5 (including title page)

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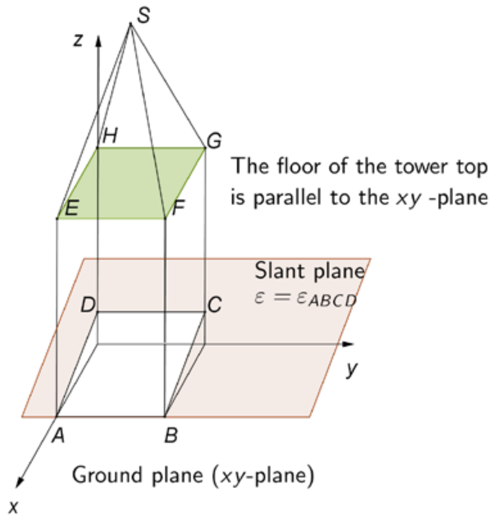
Name, Vorname Klasse Nummer

Exercise 1 **Vector Geometry**	a	b	c	d	e	f	**Points**
	1.5	1.5	1.5	2	3	3	12.5



On an alpine meadow forming a slanted plane there is a church tower with a top in form of a right square based pyramid (see figure). The vertices *A*(10|0|0), *B*(10|10|0), *C*(0|10|2), *E*(10|0|20) and *S*(5|5|30) are given.

a. Determine a parameter equation (vector equation) and a coordinate equation (Cartesian equation) of the slant plane ε.

b. Find the inclination angle of the slant plane ε (with respect to the xy-plane).

c. At a certain time of day, the sunlight shines in direction of $\vec{v} = \begin{pmatrix} -4 \\ 1 \\ -5 \end{pmatrix}$. Determine the coordinates of the point S' (on the slant plane ε) where the shadow of the apex *S* is located.

d. Determine the angle between the roof edge *ES* and the floor of the tower top.

e. Peasly *P*(215|-72|70) and Quaisley *Q*(-225|148|4) are farms near the church tower. There is a straight road ℓ connecting *P* with *Q*. Which point *T* on the road ℓ is closest to *S*?

f. Which point *R* inside the tower top is equally distant from the five vertices *E*, *F*, *G*, *H* and *S?* Give this distance as well.

Exercise 2	a	b	c	Points
Calculus	5.5	3.5	2	11

The function $f(x)=\dfrac{x^3-8x^2+20x}{2x^2-8x+8}$ is given.

a. Analyse the function $f(x)$ (domain, asymptotes, zeros, turning points (extrema), points of inflection). Draw the graph of $f(x)$ and all its asymptotes. *Unit: 2 squares or 1 cm*.

b. Calculate the area of the region enclosed by the tangent t to the graph of $f(x)$ at the point $P\left(4\,|\,f(4)\right)$, the positive x-axis and the graph of $f(x)$.

c. The line $\ell:\ \ y=\frac{1}{2}x-2$, the graph of $f(x)$ and the two vertical lines $x=3$ and $x=b$ where $b>3$ enclose a region. Express its area in terms of b. Determine the area of the region for $b\to\infty$, if it is finite.

Exercise 3	a	b	c	d	e	f	g_1	g_2	**Points**
Calculus	0.5	1	2	1	1.5	1.5	2	2.5	12

The function $f(x)$ is defined as $f(x)=(x-a)\cdot\sqrt{b-x}$ where $a,b\in\mathbb{R}$.

a. Determine the domain of $f(x)$.

b. State the conditions for a and b under which the graph of $f(x)$ has two different zeros. Under which conditions does it have just a single zero?

c. Determine the values of a and b for which the graph of $f(x)$ has the maximum point $H(4\,|\,2)$.

In the following, assume $a=0$ and $b=3$, i.e., $f(x)=x\cdot\sqrt{3-x}$.

d. Show that $f(x)$ does not have a point of inflection.

e. Calculate the angle of intersection between the graph of $f(x)$ and the x-axis in the origin.

f. A solid of revolution is formed by rotating the part of the graph of $f(x)$ between its two zeros about the x-axis. Calculate the volume of this solid.

g. The straight line ℓ_1 with positive slope passes through the origin *O* and intersects the graph of $f(x)$ in point $Q(u\,|\,f(u))$ lying in the first quadrant. The straight line ℓ_2 also passes through *Q* and is perpendicular to ℓ_1. It intersects the x-axis in the point *R*.

g_1. Find the equations of the lines ℓ_1 and ℓ_2 in terms of u.

g_2. Calculate the coordinates of *Q* for which the hypotenuse in the right-angled triangle *ORQ* has maximum length.

Exercise 4 **Probability**	a	b_1	b_2	b_3	c	**Points**
	0.5	0.5	1	1.5	1	
	d_1	d_2	e	f	g	12.5
	1	2	2	1.5	1.5	

Pass is a game that is played with a fair die against a bank. To play the game, you have to pay a stake of 20 francs.

You start the game by rolling the die. The number you get is written down.

Next roll(s): If you roll a number that has already been recorded, the game is over. Otherwise, the number is written down, you get 10 francs and you may continue rolling the die.

The game ends as soon as a number is rolled which has previously been recorded.

a. How many different outcomes (A_1: maximum win, …, A_n: maximal loss) are possible in a game of *Pass*?

b. You play 5 rounds of *Pass* and write down the sequence of outcomes, e.g. $A_4A_3A_4A_2A_1$. How many different arrangements are possible if

b1. there are no restrictions?

b2. the maximum win has appeared at least once?

b3. if there are exactly two different outcomes (e.g. $A_4A_2A_2A_4A_4$ or $A_1A_3A_3A_3A_3$)?

c. Show that the probability to get the maximum win in a game of *Pass* is $\frac{5}{324}$.

d. What is the probability that you

d1. never get the maximum win in 5 rounds?

d2. get the maximum win once or twice in 5 rounds?

e. How many rounds do you have to play at least to get the maximum win at least once with a probability of at least 95%?

f. What is the probability to win money if you play one round of *Pass*?

g. What is the expected win or loss in one round of *Pass*?