KANTON LUZERN
Bildungs- und Kulturdepartement

Kantonsschule Alpenquai Luzern

Mathematics Basic Course

Written Matura Exam 2016

Teachers	*Roman Oberholzer (roman.oberholzer@edulu.ch)*
Classes	*6c, 6d, 6m*
Date of the exam	*Friday, 20th of May, 2016*
Time	*180 minutes*
Aids allowed	*- "Mathematics Formulary", Adrian Wetzel* *- A dictionary (book, no electronic translator)* *- TI-30, Voyage 200 (or TI-92 Plus) without user manual*
Instructions	*- Importance is attached to a proper and clear representation.* *- Write each exercise on a separate sheet of paper.* *- All solutions must show the steps leading to the result.* *- Put your personal number, your name and your class on every sheet of paper.*
Maximum points per exercise	*Exercise 1: 13.0* *Exercise 2: 11.5* *Exercise 3: 9.0* *Exercise 4: 12.5* ***Total: 46.0***
Points required for a grade of 6	*41 points*
Number of pages	*5 (including the title page)*

Exercise 1 **Vector Geometry**	a	b	c_1	c_2	d	e	**Points**
	3	2	2.5	1	2.5	2	13

The line $\ell: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + t \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix}$ and the plane $\mathcal{P}$ with the Cartesian equation

$\mathcal{P}$: 2x + 3y + 7z + 37 = 0 are given.

a. The line ℓ and the plane $\mathcal{P}$ intersect each other at point S. Determine the coordinates of point S and the intersection angle between the line ℓ and the plane $\mathcal{P}$.

b. Calculate the distance of the point P(-3/-4/-1) from the plane $\mathcal{P}$.

c. R' is the reflected point of the point R(3/4/1) across the plane $\mathcal{P}$.

c_1. Find the coordinates of point R'.

c_2. Determine the equation of the line ℓ' which is obtained by the reflection of the line ℓ across the plane $\mathcal{P}$.

d. How far away is the origin O(0/0/0) from the line ℓ?

e. Furthermore, the plane $\mathcal{R}: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ k \\ -2 \end{pmatrix} + u \begin{pmatrix} -3 \\ 2 \\ 0 \end{pmatrix} + v \begin{pmatrix} 1 \\ k \\ -2 \end{pmatrix}$ is given. Find the value of k in order to prevent the planes $\mathcal{P}$ and $\mathcal{R}$ from intersecting each other.

Exercise 2 **Calculus**	a	b	c	d	**Points**
	3.5	2.5	2	3.5	11.5

A fourth order polynomial p has exactly two zeroes, one at x = 0 and one at x = 2. It reaches its maximum at $x = \frac{3}{2}$ and has an inflection point at W(1/?).

The area enclosed by the polynomial p and the x-axis measures $\frac{8}{5}$.

a. Find the equation of the polynomial p.

Solve the following exercises with the polynomial $p(x) = -x^4 + 2x^3$.

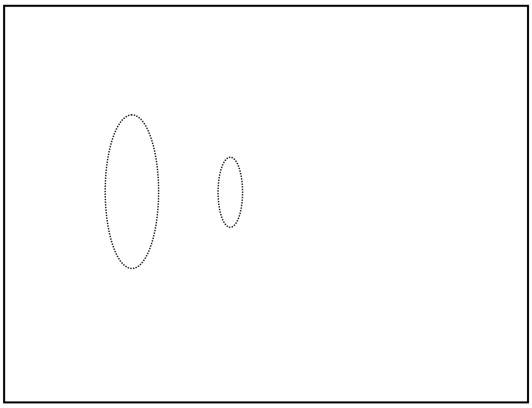
b. The non-horizontal tangent of inflection to the polynomial p, together with the polynomial p itself and the x-axis, encloses two regions. Calculate the area of the smaller region.

c. Now we will consider the function $f(x) = \frac{p(x)}{ax^k + b}$ where p(x) is the polynomial from above. Find the values of a, b and k in such a way that the line $y = \frac{1}{2}x - 1$ is an oblique asymptote and the line x = 1 is a vertical asymptote of the graph of f.

d. Going back to the polynomials, we will look at the more general polynomial $p_c(x) = -x^4 + 2x^3 - 2cx + c$. For each value of c, the graph of the polynomial p_c has two inflection points. For what value of c is the distance between these two inflection points as small as possible?

Exercise 3 Calculus	a	b	c	d	e	f	**Points**
	1.5	1	1	1.5	1.5	2.5	9

The curve $y = \sqrt{x} \cdot e^{-x^2}$ with $x \geq 0$ is given. Rotating about the x-axis, the curve forms a solid similar to a bottle that is bulged (= gebaucht) at the bottom and narrows upwards (*see figure at the right*). The units in the coordinate system are decimeters (dm).



a. What is the size of the biggest diameter of the bottle?

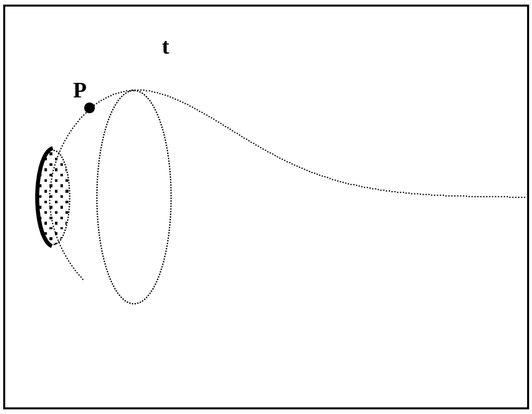
b. The bottle has a label (= Etikette). The top edge of the label is located at the height x where the curvature of the curve y equals zero. At what height x of the bottle is the top edge of the label situated?

c. Calculate the volume of the bottle which stretches to infinity to the right.

d. To the right of the inflection point of the original curve, the bottle is cut at the upper end in order to push in (= einstecken) a cork (= Korkzapfen) into the bottle. To fix the cork correctly, the absolute value of the gradient angle of the curve y has to be smaller than 5°. From what height of the bottle on is this the case?

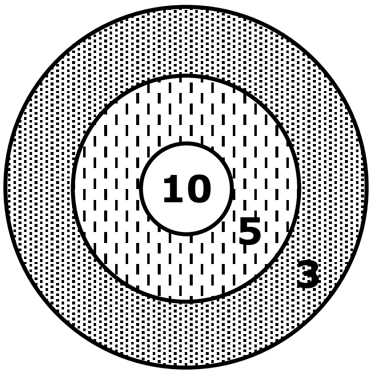
e. By what factor k, k > 0, must the graph of y be stretched (i.e. we will consider now the function $y_k = k \cdot \sqrt{x} \cdot e^{-x^2}$) so that the bottle has a volume of one liter if it is filled up to the height of x = 1.5?

f. In order for the bottle to have a circular base area and therefore a safe footing (= Stand), the tangent t is drawn at point P to the curve y. Now the solid of revolution with $x \geq 0$ is formed by the tangent t on the left of P, and by the curve y on the right of P (*see figure at the right*). Determine the coordinates of point P in such a way that the circular base area has a radius of r = 0.25 (dm).



Exercise 4 Combinatorics/Probability	a_1	a_2	a_3	a_4	b	**Points**
	0.5	0.5	1	1	1.5	
	c_1	c_2	d	e		12.5
	2.5	1.5	2	2		

On a circular dart board, two concentric circles are drawn to define three different areas for the "10", the "5" and the "3". If a dart (= Pfeil) is thrown at the board, there are four different outcomes that are possible: the dart hits one of the numbers or it misses the board.



a. There are six people. Each person throws one different-colored dart at the board. How many different possibilities for the six darts are there if

a_1. there are no restrictions?

a_2. only the first and the fourth person hits "5"?

a_3. two people in succession (= hintereinander) never have the same outcome?

a_4. "10" is hit at least five times?

The dart board has a radius of 30cm; and the radii of the two concentric inner circles measure 18cm and 6cm respectively. Paul hits the dart board with a probability of $\frac{3}{4}$, and when he strikes (= treffen) the dart board, each point on it with the same probability. Based on this, we get the following probabilities:

P[Paul misses the board] = $\frac{1}{4}$; P[Paul hits "3"] = $\frac{12}{25}$; P[Paul hits "10"] = $\frac{3}{100}$

b. Prove with the help of geometric arguments that P[Paul hits "5"] = $\frac{6}{25}$.

***Remark**: A proof using the fact that the sum of the probabilities of all events equals 1 will not be accepted.*

c. Paul throws three darts. Find the probability that he

c_1. gets at least 5 points in total.

c_2. hits all three areas, i.e. he strikes "10", "5" and "3" exactly once.

d. How many times must Paul throw a dart at least, until he hits "10" once at least with a probability of at least 99%?

e. You have to pay a stake (= Einsatz) of 4 Francs to throw a dart at the board. The number you hit on the board shows you the amount of money you win.

What is the expected win/loss of this game if one dart is thrown?