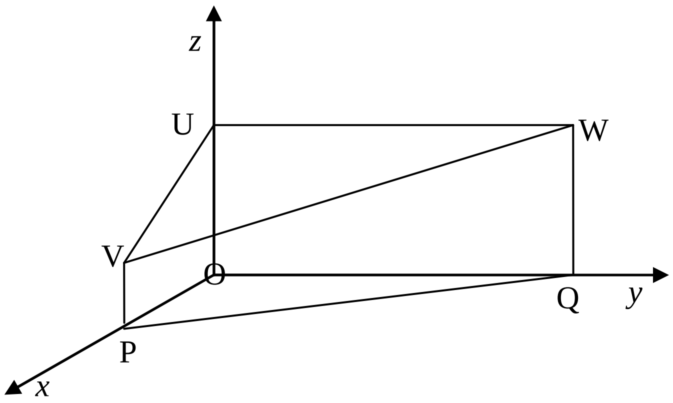
KANTON LUZERN
Bildungs- und Kulturdepartement

Kantonsschule Alpenquai Luzern

Written Matura Exam 2014

Subject	*Mathematics basic course (bilingual)*
Teachers involved	*Johanna Maluschke (johanna.maluschke@edulu.ch)* *Daniel Muzzulini (daniel.muzzulini@edulu.ch)* *Roman Oberholzer (roman.oberholzer@edulu.ch)*
Classes	*6Lc / 6Ld / 6Wc*
Date of the exam	*Friday, Mai 23, 2014*
Duration	*180 Minutes*
Media permitted	*Adrian Wetzel „Mathematics formulary"* *Calculator TI30; Voyage 200 (or TI-92 Plus), no reference book* *A dictionary (book, no electronic translator)*
Instructions	*Pay attention to a clear and readable presentation.* *Start each problem with a new sheet and put your name and number on all the sheets. Mark drafts clearly.* *The steps leading to the results are part of the answer and must be documented clearly and completely.*
Maximum points	*Problem 1: 12.0* *Problem 2: 12.5* *Problem 3: 11.0* *Problem 4: 13.0* *Total: 48.5*
Marking	*43 points are required for grade 6.*
Number of pages (including title page)	*5*

1 **Vector geometry**	a)	b)	c)	d)	e)	f)	**Points**
	2	1.5	2	1	2	3.5	12



The solid shown above represents a modern multi-purpose hall with a triangular base and an oblique roof. The back wall OQWU forms a rectangle and is contained in the *yz*-plane. Furthermore the points $P(4|0|0)$, $Q(0|12|0)$ and $V(4|0|2)$ are given.

a) Determine a vector equation and a Cartesian equation of the plane containing the trapezium PQWV.

b) The plane defined by the points U, V and W has an equation of the form $3x+4z+d=0$. Determine the value of the constant *d*, and – by making use of the equation of the plane – the coordinates of U and W.

In the following use $W(0|12|5)$.

c) Calculate the coordinates of the intersection point D of the diagonals in the trapezium PQWV.

d) Show that point $S\left(\frac{4}{3}|4|4\right)$ is the centroid (= center of gravity) of the triangle UVW.

e) Calculate the angle $\measuredangle QSP$.

f) At point S of the roof UVW there is a lamp, which sends a ray of light ℓ into the hall in a direction perpendicular to the roof. This ray hits the back wall at point R, where it is reflected. The reflected ray hits the floor of the building at point B. Determine the coordinates of the points R and B.

2 Calculus	a)	b)	c)	d)	e)	f)	**Points**
Calculus	5	1	1	1	1.5	3	12.5

The function $f(x)=\dfrac{(x+2)^2}{x^2}$ is given. Its graph is called G_f.

a) Determine the domain, the zeros, the asymptotes, the maximum and minimum points as well as the points of inflection of *f*.

b) Sketch the graph G_f and its asymptotes according to the results obtained in a).

Additionally, the lines $g_a(x)=ax-2a+4$ with the parameter *a* as well as the point $\mathrm{Q}(2\,|\,4)$ on the graph of *f* are given.

c) Show that all the lines g_a contain the point Q.

d) Determine the value of *a*, which makes the line g_a a tangent of G_f at the point Q.

e) In the sketch of exercise b), draw all the lines g_a, having exactly two points in common with the graph G_f. *There are no calculations required!*

f) For positive numbers *u*, the points $\mathrm{A}\left(u\,|\,0\right)$, $\mathrm{B}\left(u\,|\,f(u)\right)$ and the origin $\mathrm{O}\left(0\,|\,0\right)$ form a triangle ABO. What value of *u* minimizes the area of the triangle? Calculate the minimum area as well.

3 Calculus	a)	b)	c)	d)	e)	f)	**Points**
	0.5	2.5	1.5	1.5	2	3	11

The function *f* is given by $f(x)=\sqrt{x+2}$. Its graph is called G_f.

a) Determine the domain of *f*.

b) Calculate the area of the region enclosed by G_f, the tangent to G_f at $x_0=7$ and the *x*-axis.

c) Calculate the angle between the tangent to G_f in $\mathrm{P}(2\,|\,2)$ and the line through P and $\mathrm{Q}(2.5\,|\,0)$.

d) The area between the horizontal line through $\mathrm{P}(2\,|\,2)$ and G_f is rotated about the *x*-axis over the interval $[-2;\ 2]$. Determine the volume of the obtained solid of revolution.

Furthermore, the function $h(x)=\frac{1}{4}(x-u)^2+\frac{3}{4}$ depending on the parameter *u* is given. Its graph is called G_h.

e) Determine the value of the parameter *u*, for which the graphs G_f and G_h touch each other in a point B. What are the coordinates of B?

Now, instead of $h(x)$ consider the general parabola $g(x)=a(x-u)^2+v$ having the parameters *a*, *u* and *v*.

f) The graph of *g* intersects the *x*-axis twice. The slope *m* at its smaller zero $x_1=2$ is $m=6$. The area enclosed by the graph of *g* and the *x*-axis measures A = 16. Calculate the values of *a*, *u* and *v*.

4	a)	b)	c)	d)	e)	f)	**Points**
Combinatorics / Probability	2	1	2	2.5	2	3.5	13

Ms. Shywiler has the four children Antony, Bruno, Claudia and Daniela.

a) In how many different ways can the four children be seated in a row on a four-seater sofa, if

i. no restrictions are made;

ii. Daniela wants to sit in an outer position;

iii. Claudia does not want to sit next to Bruno?

b) In how many different ways can Ms. Shywiler together with three arbitrarily selected children sit on the four-seater sofa, if no further restrictions are made?

In the evenings Ms. Shywiler meets her children **A**ntony, **B**runo, **C**laudia and **D**aniela with the (independent) probabilities $P(A)=\frac{2}{5}$, $P(B)=\frac{3}{8}$, $P(C)=\frac{3}{10}$ and $P(D)=\frac{1}{5}$ at home.

c) What is the probability that on an evening

i. just Antony and Bruno are at home;

ii. at least one of the children is at home?

d) What is the probability that during a week (7 evenings)

i. there are exactly 2 evenings without any children at home;

ii. there are more than 2 evenings without any children at home?

e) How many evenings must pass by at least, until Ms. Shywiler meets all her children at home at one evening at least with a probability of at least 95%?

Shywilers like the following game of chance:

The player pays a stake of two Swiss Francs to the housekeeping money (= Haushaltskasse) and tosses a fair coin repeatedly, until one of the two sides (Heads or Tails) has occurred three times in total. The pay-out in Swiss Francs is taken from the housekeeping money and equals the number of Heads the player has tossed, which ends the game.

f) Calculate the expected win or loss of this game rounded to 5 Rappen.