KANTON
LUZERN
Bildungs- und Kulturdepartement

Kantonsschule Alpenquai Luzern

Mathematics Basic Course

Written Matura Exam 2013

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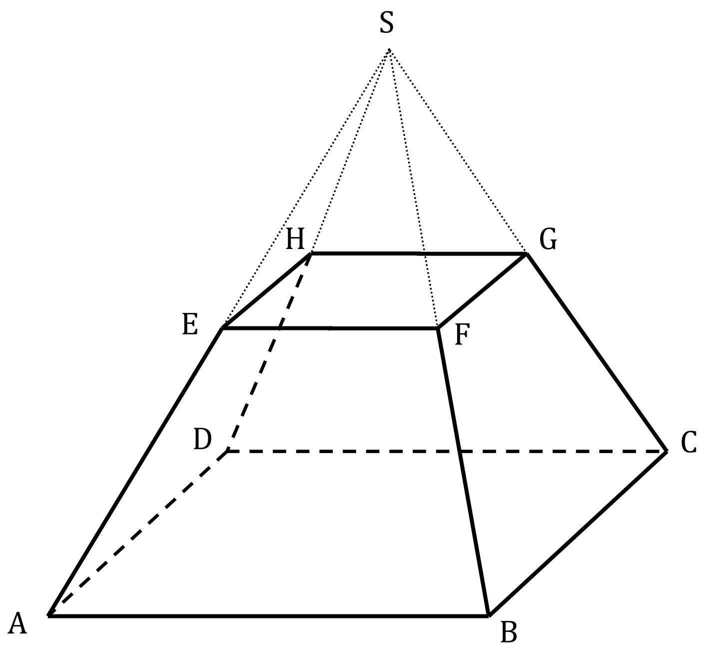
Classes	*6Lc, 6Wc*
Date of the exam	*Friday, 24th of May, 2013*
Time	*180 minutes*
Aids allowed	*- "Mathematics Formulary", Adrian Wetzel* *- A dictionary (book, no electronic translator)* *- TI-30, Voyage 200 (or TI-92 Plus) without user manual*
Instructions	*- Importance is attached to a proper and clear representation.* *- Write each exercise on a separate sheet of paper.* *- All solutions must show the steps leading to the result.* *- Put your personal number, your name and your class on every sheet of paper.*
Maximum points per exercise	*Exercise 1: 13* *Exercise 2: 14* *Exercise 3: 11* *Exercise 4: 11.5* ***Total: 49.5***
Points required for a grade of 6	*42 points*
Number of pages	*5*

Exercise 1 - Vector Geometry	a	b	c	d	e	f	Points
	1.5	1.5	2	3.5	2	2.5	**13**

At the half of its height, a right pyramid ABCDS is intersected by a plane which is parallel to its base ABCD (see figure below). The resulting frustum of a pyramid (= Pyramidenstumpf) is defined by the points A(-3/11/-3), B(5/3/-7), C(13/7/1), D(5/15/5), E(-2/4/4), F(2/0/2), G and H.



a. Prove that the base ABCD of the pyramid is a square.

b. Determine the Cartesian equation of the plane $\mathcal{P}_{ABC}$ through the points A, B and C.

c. Calculate the angle between the edge AE and the base ABCD.

d. Find the coordinates of the points P on the line through the points C and E which have a distance of $3\sqrt{3}$ from point F.

e. Determine the coordinates of the apex (= Spitze) S of the original pyramid.

f. Calculate the distance of the point E from the plane $\mathcal{P}_{ABC}$.

Exercise 2 - Calculus	a	b	c	d	e	Points

	2	4.5	1	2.5	4	**14**

The functions $f(x)=\dfrac{x^2+a}{2x+b}$, a < 2 and b < 2, and $g(x)=e^{x-2}$ are given.

a. The graph of the function f intersects the graph of the function g at x = 2. Furthermore, the tangent to the graph of f at x = -1 is parallel to the line $\ell: 2x+y+4=0$. Find the values of a and b.

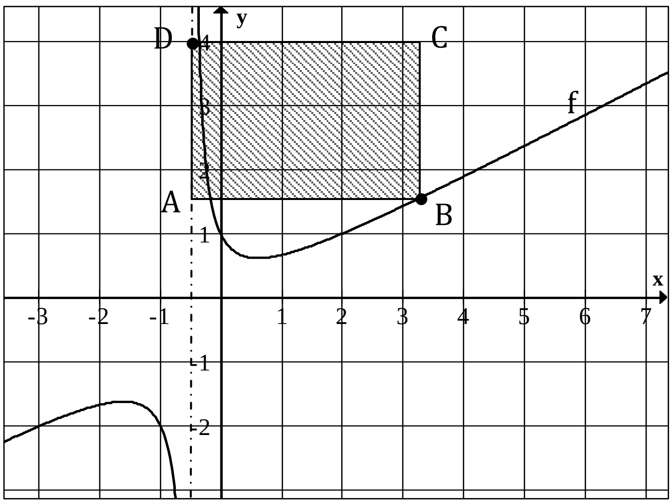
Solve the following exercises with the function $f(x)=\dfrac{x^2+1}{2x+1}$.

b. Determine the domain, the zeros, the stationary points, the inflection points and the asymptotes of the graph of f. *The graph of the function is not required.*

c. Calculate the angle of intersection of the graphs of f and g at their intersection point S(2/?).

d. Calculate the area enclosed by the graph of g, the tangent t to the graph of g at S and the x-axis.

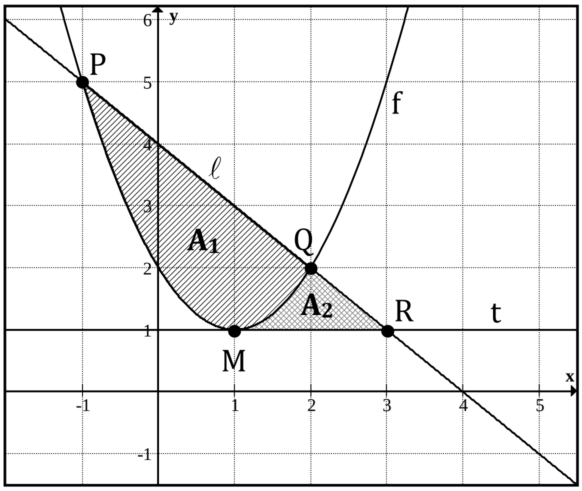
e. Starting at point D(-0.5/4), the rectangle ABCD is drawn into the coordinate system in such a way that its sides are parallel to the axes and point B lies on the graph of f below point C in the first quadrant (see figure at right). Determine the coordinates of B in order for the area of the rectangle to be a maximum. Calculate this maximum area as well.



Exercise 3 - Calculus	a	b	c	d	e	f	**Points**

	1	3	2.5	1.5	1.5	1.5	**11**

The figure shows the parabola $f(x) = (x-1)^2 + 1$, the line $\ell(x) = -x + 4$ and their intersection points P and Q. M is the low point of the parabola f, and t is the tangent to the graph of f at M.



a. Calculate the area A_1 which is enclosed by the parabola f and the line ℓ.

b. The parabola f, the line ℓ and the tangent t enclose the area A_2 to the right of M in the first quadrant. This area rotates about the x-axis. Calculate the volume of the resulting solid of revolution.

c. Determine the equation of the line h which is parallel to the line ℓ and which encloses, together with the parabola f, an area of $\frac{4}{3}$.

d. Prove that the triangle PMQ is a right-angled triangle with the right angle at Q.

e. Determine the proportion between the areas of the triangle PMQ and the area A_1 (of exercise a.).

f. A line k, with slope m and passing through the point Q, intersects the parabola f at point S for the second time. Prove that the x-coordinate of S equals the slope m of the line k.

Exercise 4 - Probability	a_1	a_2	a_3	b_1	b_2	**Points**

	0.5	0.5	1	0.5	1	
	b_3	c	d	e_1	e_2	**11.5**
	1	3	2	1	1	

At the market, Mr Brown, together with his two daughters Mia and Audrey, and Ms Simpson, together with her three sons David, Brian and Nicholas are queuing in front of a fortune wheel. There are no other people waiting in the queue.

a. How many different possibilities of queuing are there if
   - a_1. there are no restrictions;
   - a_2. the Brown family stands in front of the Simpson family;
   - a_3. all the children want to queue one after the other?

The fortune wheel is divided into twelve sectors of equal size, but different colors: two sectors are green, two are yellow, three are blue and five are red. If the wheel is spun (= gedreht) at random, one sector is indicated by stopping under the pointer.

b. If the fortune wheel is spun four times, find the probability that
   - b_1. a red sector is indicated four times;
   - b_2. a red sector is indicated at the fourth spin for the first time;
   - b_3. a green, a blue, a yellow and a red sector are each indicated once.

A charity organization offers the following game: For a stake (= Einsatz) of 5 francs, the fortune wheel can be spun four times. If the indicated sector is red four times, the player wins a first prize (=Hauptpreis) of 100 francs. If there are exactly three red sectors indicated in four spins, the player gets a booby prize (= Trostpreis) of 5 francs. In all other cases, the player wins nothing.

c. The game serves to generate donations for the charity organization which hopes to take 1 franc per game on average. Is the expectation of the organization correct or not? Justify your answer by a calculation.

d. Ms Smith wants to take a first prize home for her daughter. How much money at least must she take with her to the market place in order to win at least one first prize with a probability of at least 95%?

e. At a different market place, the fortune wheel mentioned above is spun seven times. Find the probability that
   - e_1. a blue sector is indicated exactly three times;
   - e_2. a red sector is indicated at least five times.