Kantonsschule Alpenquai Luzern

Written Matura Exam 2012 *Mathematics Basic Course*

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Classes	*6Lc, 6Wd*
Date of the exam	*Tuesday, 29th of May, 2012*
Time	*180 minutes*
Aids allowed	- *“Mathematics Formulary”, Adrian Wetzel* - *A dictionary (book, no electronic translator)* - *TI-30, Voyage 200, without user manual*
Instructions	- *Importance is attached to a proper and clear representation.* - *Write each exercise on a separate sheet of paper.* - *All solutions must show the steps leading to the result.* - *Put your personal number, your name and your class on every sheet of paper.*
Maximum points per exercise	*Exercise 1:* 14 *Exercise 2:* 10 *Exercise 3:* 11 *Exercise 4:* 10 *Total:* 45
Number of pages	*4*

Exercise 1 – Calculus	a	b	c	d	e	**Points**
	3	5	2	2	2	14

A third order polynomial f(x) has a stationary point (= an extremum) at x = -2 and the origin lies on the tangent at the graph of f(x) at the point of inflection W(-1/2). Furthermore, let $g(x)=\dfrac{2x^2+x+11}{3-3x}$.

a. Determine the function equation of the polynomial f(x).

If you couldn't solve exercise a., continue with $f(x)=\frac{2}{3}x^3+2x^2+\frac{2}{3}$.

b. The function g(x) has neither zeros nor points of inflection. Determine the derivatives, domain, stationary points (= extrema) and asymptotes of g(x) and then draw the graph of the function. *Units*: 2 squares or 1cm.

c. For which values of x do the graphs of f(x) and g(x) intersect in a right angle?

d. Determine the area that is enclosed by the graphs of f(x) and g(x) and the y-axis.

e. By how many units does the graph of f(x) have to be translated vertically up or down so that it touches the oblique asymptote of the graph of g(x)?

If you couldn't find the oblique asymptote of g(x) at b., solve exercise e. with the oblique asymptote $y=-\frac{3}{4}x-1$.

Exercise 2 – Calculus	a	b	c	d	**Points**
	2	1.5	3	3.5	10

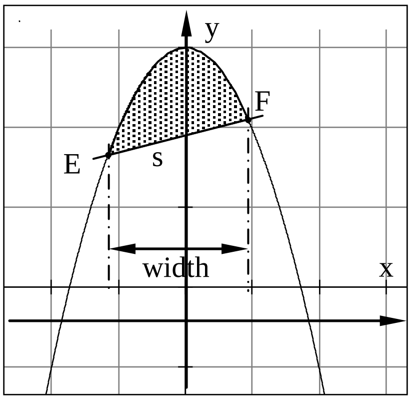
Let $f(x)=-x^2+3$ and $g(x)=e^x$.

a. At which point does the tangent at the graph of g(x) at the point A(0/y) intersect the graph of f(x)?

b. The area that is enclosed by the graphs of f(x) and g(x) is rotated about the x-axis. Calculate the volume of the solid of revolution.

c. The graph of the parabola f(x) encloses with the x-axis an area. Into this area a triangle BCD with B as negative zero of f(x), C(u/0) and D(u/f(u)) is inscribed. For which point D is the area of the triangle maximal?

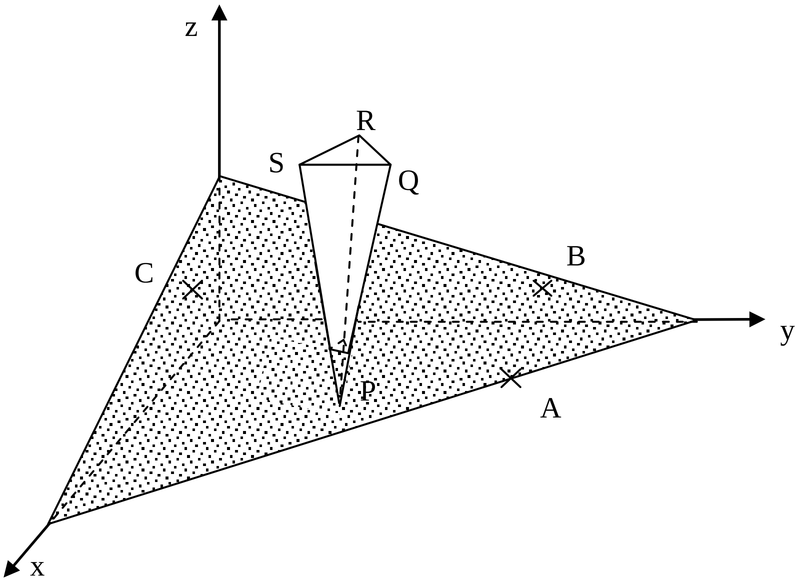
d. A secant s through the two points E and F on the parabola encloses with the parabola the shaded area, a so called **parabolic segment** (see figure). The difference of the x-coordinates of the points E and F is called **width of the parabolic segment**. Show that all segments with width 2 of the parabola $f(x)=-x^2+3$ have the same area.



Exercise 3 – Vector Geometry	a	b	c	d	e	Points
	2	2	1.5	2	3.5	11

A flower vase made of concrete (=Beton) in the form of an open triangular pyramid is set up at a hillside. The points A(6|12|0), B(8|7|1) and C(2|1|4) lie on the hillside, the vase is defined by the points P(6|5|-1), Q(9|6|5), R(5|9|5) and S(6|2|5), where P is in the ground.

Sketch:



a. Calculate the Cartesian equation of the hillside-plane E_{ABC}.

If you couldn't calculate the equation of the plane in exercise a., continue with the equation:
E: $3x + 4y + 14z - 67 = 0$.

b. Determine at which angle the hillside is slanted (= geneigt) with respect to the horizontal plane (= xy-Ebene).

c. Show that the angle $\alpha = \sphericalangle(SQR)$ is a right angle.

d. Before the vase could be decorated with flowers, it was filled with water during heavy rainfall. Find the volume of the water in the pyramid PQRS.

e. Although the vase stands partially in the ground, it was blown over by the wind. As fortification, the point R of the vase shall be connected with a steel rope of minimal length to an iron rail, which passes through the points B and C on the hillside. Calculate the length of this steel rope and the co-ordinates of point T on the line segment BC, where the rope is attached.

Exercise 4 – Probability	a	b	c	d	e	f	**Points**
	1	1.5	1.5	2	2	2	10

The students often use the tramway during their study trip to Prague. The price of a ticket is 12 Czech crowns. The chance of encountering a ticket control is 5%. The fine for riding the tramway without a valid ticket is 400 Czech crowns.

a. There are eight very interesting museums to visit in Prague, but the class has only time to see five of them. How many different possibilities are there to select five out of the eight museums?

b. The students are accommodated in rooms of three. In how many different ways can the 9 boys of the class be distributed to three rooms?

c. Lena buys a tramway ticket with a probability of 0.25. What is the probability p that she has to pay a fine of 400 crowns on one tramway ride?

If you couldn't solve c., continue with p = 0.03.

d. Lena rides the tramway 20 times. What is the probability that she has to pay a fine exactly twice?

e. Consider X as Lena's cost for a ride on the tramway. Determine the average cost E(X) for one ride.

f. At what minimal cost of a ticket would Lena's strategy be favorable?