KANTON LUZERN
Bildungs- und Kulturdepartement

Kantonsschule Alpenquai Luzern

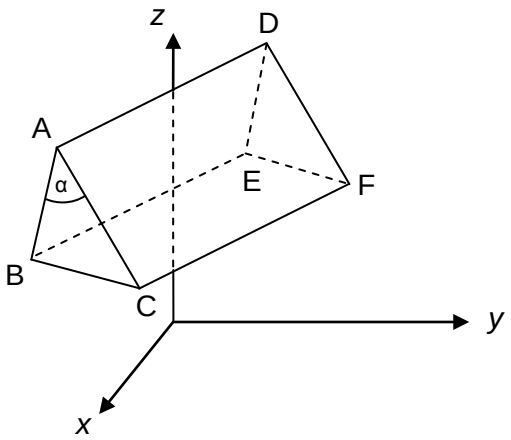
Written Matura Exam 2011

Subject	*Mathematics basic course (bilingual)*
Teachers involved	*Lukas Fischer (lukas.fischer@edulu.ch)* *Daniel Muzzulini (daniel.muzzulini@edulu.ch)*
Classes	*6Lb / 6Lc*
Date of the exam	*May 23, 2011*
Duration	*180 Minutes*
Media permitted	*Adrian Wetzel „Mathematics formulary"* *Calculator TI30; Voyage 200 (or TI-92 Plus), no reference book* *A dictionary (book, no electronic translator)*
Instructions	*Pay attention to a clear and readable presentation.* *Start each problem with a new sheet and put your name and number on all the sheets. Mark drafts clearly.* *The steps leading to the results are part of the answer and must be documented clearly and completely.*
Maximum points	*Problem 1: 11* *Problem 2: 7* *Problem 3: 7* *Problem 4: 6* *Problem 5: 10* *Total: 41*
Marking	*36 points are required for grade 6.*
Number of pages (including title page)	*4*

		a	b	c	d	e	f	g	Points
Problem 1	**Vector geometry**	0.5	1	2	1.5	3	1	2	**11**

Given the points A(-1|5|3), B(-1|2|-3), C(3|7|-1) and D(5|-1|6). The two faces ABC and DEF of the prism ABCDEF are parallel and congruent triangles.

Sketch:



a) Calculate the coordinates of point F.

b) Show that the triangle ABC is isosceles.

c) Determine the Cartesian equation of the plane ε containing the points A, B and C.

d) Calculate the angle $\alpha = \sphericalangle(\mathrm{BAC})$.

e) i) Which point on the line AD is closest to the origin?
ii) Is this point on the edge $\overline{\mathrm{AD}}$?

f) Show that the face ABC is perpendicular to the lateral edges.

g) Determine the volume of the prism ABCDEF.

		a	b	c	Points
Problem 2	**Calculus**	4	1	2	**7**

a) Determine the equation of the third order polynomial $f(x)$ with the following properties: The graph of $f(x)$ intersects the *y*-axis at $\frac{11}{3}$ and touches the line g: $y = \frac{1}{2}x - 3$. Furthermore, it has a point of inflection at $x = 2$ with a tangent of inflection perpendicular to g.

If you couldn't solve a) use the substitute $\tilde{f}(x) = \frac{5}{24}x^3 - \frac{15}{8}x^2 + \frac{29}{8}x + \frac{53}{24}$ instead of $f(x)$ in b) and c), which has its point of inflection at $x = 3$.

b) Give the equation of the tangent of inflection *t*.

c) Calculate the area that is enclosed by the graph of the function $f(x)$ and the line g.

		a	b	c	Points
Problem 3	**Calculus**	3	2	2	**7**

Given the family of functions $f_a(x) = \dfrac{3 \cdot (x-2)^2 \cdot (x-a)}{a \cdot (x^2-9)}$ where $a \neq 0$.

The tasks are independent of one another.

a) i) Show that all functions $f_a(x)$ have the same *y*-intercept.

ii) For which value of a is the *y*-intercept of the slant asymptote of $f_a(x)$ equal to –6?

iii) Determine the number of vertical asymptotes of $f_a(x)$ in terms of a.

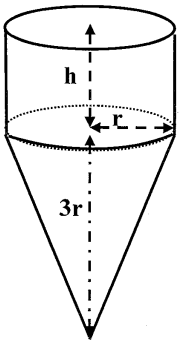
In the following let $a = 3$:

b) Determine the coordinates of the turning points (extrema) of $f_3(x)$.

c) The line h: $y = \frac{1}{4}$, the graph of $f_3(x)$ and the *y*-axis enclose a triangle-like shape that is rotated about the *x*-axis. Calculate the volume of this solid of revolution.

		Points
Problem 4	**Calculus**	**6**

An ice-cream producer wants to gain market shares with a new type of cornet. Its shape should be as shown:



Lower part: right circular cone whose base radius r is equal to one third of the height.

Upper part: right circular cylinder of the same radius r.

The cornet will have a total volume of 150 cm^3. Furthermore, its dimensions are chosen in such a way that the total lateral area (cone plus cylinder) is minimized.

Calculate the radius r and the height of the cornet.

	a i	ii	iii	iv	b i	ii	iii	iv	**Points**
Problem 5 Probability	0.5	1.5	1	1.5	1	1	1.5	2	**10**

Petra has passed the Matura exam and gets an iPod from her parents. Her first download consists of four songs each by *Queen*, *Take That*, *Gotthard* and eight songs by the *Beatles*.

First, she plays the 20 songs in a random order.

a) i) How many ways are there to play the 20 songs?

ii) Petra lists the order in which the songs are played. How many different lists can occur if she only picks the name of the band?

iii) Calculate the probability that the first four songs are those by *Queen*.

iv) What is the probability that at position five, a song by *Gotthard* is played for the first time?

There is a special random function on Petra's iPod which chooses the songs independently of the previous songs, so that any song can be played more than once. Now she tries out this feature.

b) i) How many ways are there to play the first 10 songs?

ii) Calculate the probability that all of the first four songs are by the *Beatles*.

iii) If seven songs are played, what is the probability that more of them are by the *Beatles* than by others?

iv) When Petra is jogging she likes listening to music. She can do one kilometer per song. How long should her run be if she wants to hear at least one entire song by the *Beatles* with a probability of at least 96%?