Kantonsschule Luzern

Written Matura Exam 2010 — *Mathematics Basic Course*

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Classes	*6Lb, 6Lc, 6Ld*
Date of the exam	*Tuesday, 25th of May, 2010*
Time	*180 minutes*
Aids allowed	- *Formula Book "Mathematical Formulas for Economists", Springer* - *Mathematical Handbook of Formulas (additional material, yello)* - *"Mathematics Formulary", Adrian Wetzel* - *A dictionary (book, no electronic translator)* - *TI-30, Voyage 200, without user manual*
Instructions	- *Importance is attached to a proper and clear representation.* - *Write each exercise on a separate sheet of paper.* - *All solutions must show the steps leading to the result.* - *Put your personal number, your name and your class on every sheet of paper.*
Maximum points per exercise	*Exercise 1: 11* *Exercise 2: 12* *Exercise 3: 15* *Exercise 4: 10* *Total: 48*
Number of pages	*4*

Exercise 1 – Calculus	a	b	c	d	Points
	2	2	3	4	11

The functions $f(x) = \frac{x^3}{x^2 - 4}$ and $g(x) = \frac{1}{e} e^x$ are given.

a. Determine all asymptotes of the function f(x).

b. Prove that the oblique asymptote of f(x) and the graph of g(x) touch each other.

c. The graph of g(x), the oblique asymptote of f(x) and the x-axis enclose an area that stretches to infinity. This area is rotated about the x-axis. Calculate the volume of the generated solid of revolution.

d. Let H be the maximum point and L the minimum point of the graph of f(x). Find the coordinates of point P on the graph of g(x) so that the triangle HLP is isosceles with $\overline{HP} = \overline{LP}$.

Exercise 2 – Calculus	a	b	c	Points
	5	3	4	12

The graph of a third order polynomial f(x) passes through the origin and has a minimum point L on the positive x-axis. The maximum point H has 16 as its y-coordinate. The x-axis and the graph of f(x) enclose an area A = 27.

a. Determine the equation of f(x) as well as the coordinates of L and H.

Solve the following exercises with the function equation $f(x) = 4x \cdot (x-3)^2$

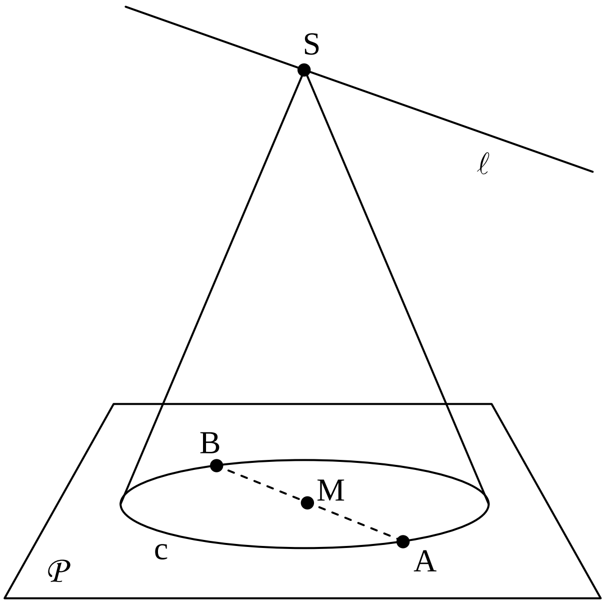
b. L is the minimum point of the graph of f(x). The line ℓ through the points S(2/f(2)) and A(a/0) with 0 < a < 2 together with the x-axis and the graph of f(x) enclose the region ALS with an area A = 9. Find the value of a.

c. The points P(u/f(u)), Q(u/0), R(2/0) and S(2/f(2)) define a trapezium PQRS which is inscribed into the area enclosed by the graph of f(x) and the x-axis in the first quadrant. Determine the coordinates of P in such a way that the area of the trapezium PQRS is as large as possible.

Exercise 3 – Vector Geometry	a	b	c	d	e	f	**Points**
	1	4	2	2.5	2.5	3	15

In the right circular cone shown in the diagram, the points A(-2/5/8) and B(6/1/0) are end-points of a diameter of the base circle c. The vertex (= Spitze) S lies on the line

$$\ell: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 8 \end{pmatrix} + t \cdot \begin{pmatrix} -2 \\ 3 \\ -1 \end{pmatrix}.$$



a. Show that point A does not lie on line ℓ.

b. Find the coordinates of the vertex S.

Now continue with S(5/-3/10).

c. Determine the Cartesian equation of the plane $\mathcal{P}$ containing the base circle c.

d. Calculate the volume of the cone.

e. Determine the angle between a slant height (= Mantellinie) of the cone and the plane $\mathcal{P}$.

f. A sphere is inscribed into the cone. Calculate its radius.

Exercise 4 – Probability	a	b	c	d	Points
	3	2	2	3	10

Anna rolls a regular die, and Beata rolls a die at which the face with 3 points has been replaced by a face with 5 points. They play the following game:

Anna and Beata roll their die once. If the numbers of points on both dice are different, the player with the lower number of points pays 3 Francs to the player with the higher number of points, and the game is over.

In the case of equal numbers of points in the first roll, Anna and Beata roll their die a second time. If the numbers of points on both dice are different now, the player with the lower number of points pays 6 Francs to the player with the higher number of points. If the numbers of points are equal again, Beata pays 20 Francs to Anna, and the game is over which means that the game ends at the latest after the dice being rolled twice.

a. Find the probability that Anna wins the game.

b. Show by calculating Anna's expected win that the game is favorable for Anna in the long run.

In the following assume that Anna wins the game with a probability of 44.91%.

c. Find the probability that Anna wins at least 3 out of 5 games.

d. How many games must be played at least for the probability to be at least 90% that Anna wins 20 Francs at least once?

Written Matura Exam 2010 (Answers) **_Mathematics Basic Course_**

1. a. vertical asymptotes: $x = 2$ and $x = -2$ oblique asymptote: $a(x) = y = x$

1. b. $a(1) = g(1)$ and $g'(1) = a'(1) = 1$

1. c. intersection point: S(1/1), $V = \frac{\pi}{6}$

1. d. P(-0.378/0.252)

2. a. $f(x) = 4x^3 - 24x^2 36x$ L(3/0), H(1/16)

2. b. $a = 0.5$

2. c. P(0.524/12.854)

3. a. Point A does not satisfy the equation of the line for one value of t

3. b. S(5/-3/10)

3. c. $\mathcal{P}: x - 2y + 2z - 4 = 0$

3. d. $V = 108\pi$

3. e. $\approx 56.31°$

3. f. $r \approx 3.21$

4. a. $\approx 44.91\%$

4. b. expected value of Anna's win = $\frac{1}{9} > 0$

4. c. $\approx 40.52\%$

4. d. 82 games