Time	180 minutes
Aids	Formula Book "Mathematical Formulas for Economists", Springer Calculator TI-voyage200 or TI-92plus (without users manual) Dictionary (book)
Maximum of points	44 points out of 5 exercises
	Grade 6 for 40 points

Your work leading to the solution has to be documented in an understandable way.

Neat, orderly and clear work is greatly appreciated.

Please solve every exercise on a new sheet of paper.

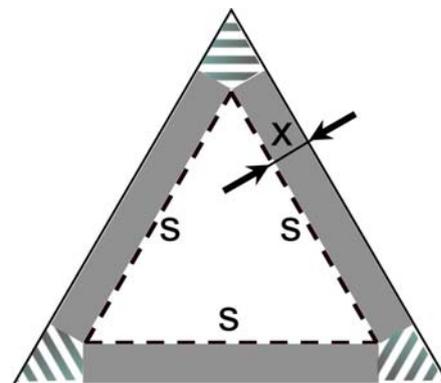## Exercise 1   5 points

A 4th order polynomial f has a point of inflection at W(1 | 2). The normal n at the point of inflection intersects the graph of the function f again at point P(−1 | 4). The area that is enclosed from below by the graph of the function f and from above by the normal n has a size of $\dfrac{14}{5}$.

Determine the function equation of f.

Exercise 2   7 points

The shaded quadrilaterals at the edges of an equilateral cardboard triangle with a side-length of 1 m are cut out. Afterwards the grey rectangles are folded along the dashed line – the result is a box in form of a right prism with height x.
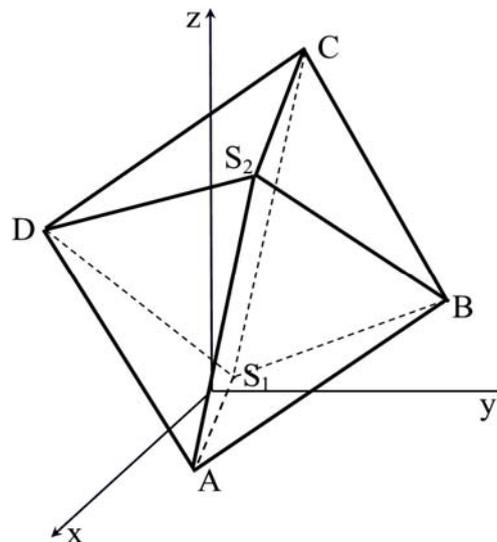

a) Show that the side-length s (sketch at the right-hand side) can be expressed as $s = 1 - 2 \cdot \sqrt{3} \cdot x$.

b) For which height x does the box reach its maximum volume?

Exercise 3   11 points

The points A(6 | 3 | 1), B(2 | 7 | 3), C(0 | 3 | 7), D, S$_1$(−1 | 1 | 0) and S$_2$ determine as vertices a right double pyramid with the square ABCD as common base (image at the right-hand side).
Both single pyramids have heights of the same length.

a) Determine the coordinates of the points D and S$_2$.

b) There is exactly one point P on the line segment $\overline{AC}$ that lies in the plane E: 19 x − 32 y + 12 z = 16.
Determine the coordinates of point P.

c) Determine the angle that is formed by the areas of the two triangles ABS$_1$ and BCS$_1$.

d) Determine the distance of point B from the line (CS$_1$).

Exercise 4   11 points

Given are the two functions $f(x) = \dfrac{x^3 - 4}{4(x-2)^2}$ and $g(x) = \dfrac{x^2 + ax + 4}{4(x-b)}$ with $a, b \in \mathbb{R}$.

a) Discuss the function f (i. e. find zeros, extrema, points of inflection and asymptotes) and draw its graph in a suitable coordinate system.

b) The area that is enclosed by the graph of the function f, the graph of its skew asymptote and the two coordinate axes in the first quadrant is rotating about the x-axis. Calculate the volume of this solid of revolution.

c) Determine the parameters a and b of the function g so that both f and g have the same asymptotes.

Exercise 5   10 points

Frank has ordered books at three bookstores. The books lie – each one wrapped in special paper in the color of the store – on the floor of his room. There are

6 books written by Agatha Christie, each one wrapped in red paper,

4 books written by Patricia Highsmith, each one wrapped in yellow paper

8 books written by Donna Leon, each one wrapped in blue paper.

Frank picks up 3 books at random.

a1)   What is the probability that the three books are by the same author?

a2)   What is the probability that he picks up exactly one book by each author?

a3)   What is the probability that he has picked up at least one book by Donna Leon?

Frank puts the books – still wrapped – next to each other onto one bookshelf.

b)   How many color patterns are there, if all books fit onto the shelf?

Afterwards, Frank unwraps the books.

c1)   How many possibilities are there now to place the books next to each other on the shelf?

c2)   How many possibilities are there to place the books, if he groups the books by authors?

Frank closes his eyes and picks 6 out of these 18 books to take on his holiday trip.

d1)   What is the probability that there are at most 2 books written by Agatha Christie?

d2)   Frank realizes that he has picked books by only two authors, and he has picked three books by each of them. What is the probability that Patricia Highsmith is one of the two authors?