KANTON LUZERN Kantonsschule Luzern

MATURA 2006
Mathematics
Basic Course

Obergymnasium

Class 6Lc (obr)

Time: 180 minutes

Only the four best solved exercises are valid for points. Each exercise gives a maximum of 10 points. Four complete answers, which must be worked out in detail, are required for a mark of 6.

Aids: Formula Book "Mathematical Formulas for Economists", Springer
A dictionary (book or electronic translator)
TI-92 or Voyage 200 with the user manual
The use of the aids is to be declared clearly.

Exercise 1

The plane $\mathcal{P}_1: x + 2y + 2z = 10$ and the coordinate planes define a pyramid S_1. The pyramid is intersected by the plane $\mathcal{P}_2$, passing through the points $A(0/0/4)$, $B(3/0/2)$ and $C(2/1/2)$.

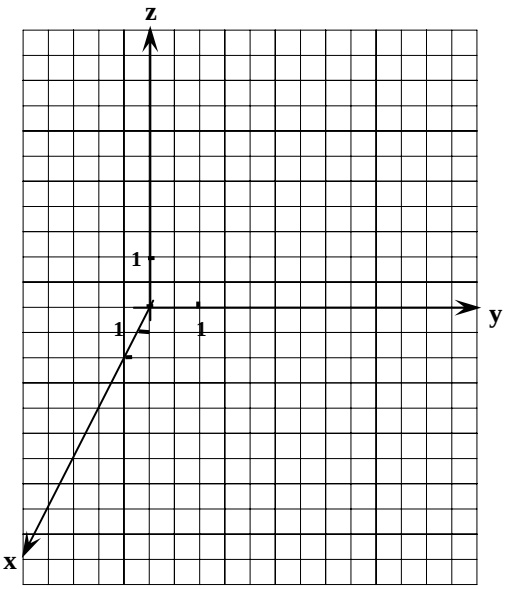
a) Find the traces (intersection lines with the coordinate planes) of the plane $\mathcal{P}_1$.

b) Sketch the pyramid in a coordinate system as shown in the figure.

c) Determine a Cartesian equation for the plane $\mathcal{P}_2$ and a vector equation for the line in which the planes $\mathcal{P}_1$ and $\mathcal{P}_2$ intersect.

d) Sketch also the plane $\mathcal{P}_2$ together with the intersection line in the same coordinate system of exercise b). The plane $\mathcal{P}_2$ and the coordinate planes define a pyramid S_2. Calculate the volume of the intersection solid $S_1 \cap S_2$.

e) Calculate the angle $\beta = \sphericalangle(ABC)$.



Exercise 2

For each $a > 2$, the function $f_a(x) = \frac{x}{a-2}\left(a - x^2\right)$ is given.

a) For a = 3, determine the symmetry, the zeros, the stationary points and the inflection points of the function $f_a(x)$, and sketch the graph.

b) In general: Find the coordinates of the maximum point H_a of the graph of f_a.

c) For what $a > 2$ is the maximum point H_a of exercise b) located at the lowest?

$\left(\text{If b) could not be solved, take } H_a\left(\sqrt{\frac{a}{3}} \Big/ \frac{a^{\frac{3}{2}}}{a-2}\right) \text{ to solve c)}\right)$

d) For what $a > 2$ is the maximum point H_a closest to the origin?

Exercise 3

A parabola $f(x)$ of the fourth degree is symmetrical with respect to the y-axis. The tangent of inflection in $A(1/-2)$ has a slope of -4.

a) Determine the function equation $y = f(x)$.

(If you could not solve exercise a), use the function $f(x) = \frac{1}{32}x^4 - \frac{3}{4}x^2 + \frac{1}{2}$, which, in contrast to the function above, has a tangent of inflection of slope -2 in point $B(2/-2)$, to solve the following exercises.)

b) The line ℓ passes through the minimum points of the graph of f. Calculate the area enclosed by the graph of f and the line ℓ.

c) The segment of the curve that lies between the positive zeros is rotated about the x-axis. Calculate the volume of the solid of revolution.

d) The two tangents of inflection and the connection line of the inflection points define a triangle. In what proportion does the graph of function f divide the area of this triangle?

Exercise 4

An employee of a travel agency knows from many years of experience that 30% of his customers book a vacation trip to the island F.

a) Find the probability that there are

a_1) exactly 15

a_2) at least 12

bookings for the island F among the next 50 reservations.

b) How many bookings must be carried out at least for the probability to be more than 99% that at least one reservation has the island F as its destination?

c) Because of the customers' booking habits, the travel agency organizes the vacation trip to the island F itself. For the flight, the travel agency charters a plane with 117 seats.
Because 8% of the passengers normally cancel their booking just a few days before the trip, the travel agency sold 125 tickets for the flight.

c_1) Find the probability that more than 117 persons show up at the airport.

c_2) What is the probability that there are still free seats on the flight?

c_3) Use the definition of the expected value to calculate how many of the 125 passengers can be expected to begin the trip?

Exercise 5

Solve the independent quizzes:

a) Under which angle do the graphs of the functions $f(x) = e^{4x}$ and $g(x) = e^{-2x}$ intersect?

b) Determine z so that the lines ℓ_1 and ℓ_2 intersect each other. Find then a vector equation of one of their angle bisectors:

$$\ell_1: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ 3 \end{pmatrix} + t \cdot \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \qquad \ell_2: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ z \end{pmatrix} + s \cdot \begin{pmatrix} -2 \\ 3 \\ 6 \end{pmatrix}$$

c) A box contains 8 white and 6 black numbered balls. If 5 balls are drawn at random without replacement, determine the probability that

c_1) exactly 2 balls are black.

c_2) at least 3 balls are white.