KANTON LUZERN
Bildungs- und Kulturdepartement

Kantonsschule Alpenquai Luzern

Written Matura Exam 2022

Subject	Mathematics Basic Course
Teacher	Roman Oberholzer roman.oberholzer@edulu.ch
Class	G18l
Date of the exam	Friday, 20th of May, 2022
Time	180 minutes
Aids allowed	- "Mathematics Formulary", Adrian Wetzel - A dictionary (book, no electronic translator) - TI-30X Pro Multiview or MathPrint (no handbook)
Instructions	- Importance is attached to a proper and clear representation. - Write each exercise on a separate sheet of paper. - All solutions must show the steps leading to the result. - Put your personal number, your name and your class on every sheet of paper.
Maximum points per exercise	Exercise 1: 8 Exercise 2: 12 Exercise 3: 13 Exercise 4: 12 **Total: 45** 38 points are required for a grade of 6.
Number of pages	5 (including title page)

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Surname, First name Class Number

Exercise 1 **Calculus I**	a	b	c	d	**Points**
	0.5	4.5	0.5	2.5	8

We consider the function $f(x)=\dfrac{-x^2+5x-4}{x}$ with the derivatives

$$f'(x)=\frac{-x^2+4}{x^2} \qquad f''(x)=\frac{-8}{x^3} \qquad f'''(x)=\frac{24}{x^4}$$

and the zeroes $Z_1(1/0)$ and $Z_2(4/0)$.

a. Show that the given first derivative f' is correct by differentiating the function f once and simplifying it.

b. Determine the domain, stationary points (maximum and minimum points), points of inflection and asymptotes of f and then draw the graph of the function f for $-10 \le x \le 10$. *Units: 1 squares or 1cm.*

c. *Calculator allowed*: Determine the area of the region under the curve of f between its two zeroes Z_1 and Z_2.

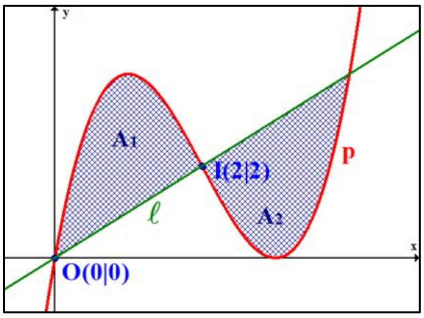
d. The point P(u/v) lies on the graph of f in the first quadrant, and O(0/0) is the origin. Find the coordinates of point P in such a way that the right-angled triangle, with the hypotenuse OP and one side lying on the x-axis, has the largest possible area.

Exercise 2 **Calculus II**	a	b	c	d	e	f	**Points**
	3.5	2	1.5	1	1.5	2.5	12

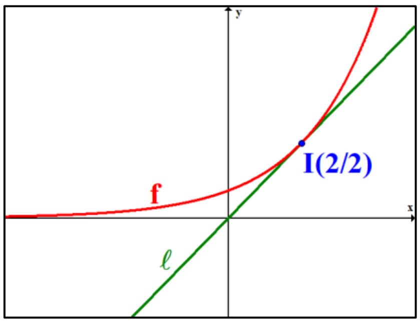
a. The graph of a third order parabola p passes through the origin and touches the x-axis at x = 6. The area under the graph of p in the first quadrant measures A = 12. Determine the function equation of the polynomial p.

Continue in the following exercises with the parabola $p(x) = x^3 - 6x^2 + 9x$.

b. The line ℓ passes through the origin O(0/0) and the inflection point I(2/2) of the parabola p (*see diagram at the right*). The line ℓ and the parabola p enclose two regions in the first quadrant. Using integral calculus, prove that these two areas are of equal size.



c. Show that the graph of the function $f(x) = 2 \cdot e^{\frac{x-2}{2}}$ has the line ℓ as its tangent line at the point I(2/2).

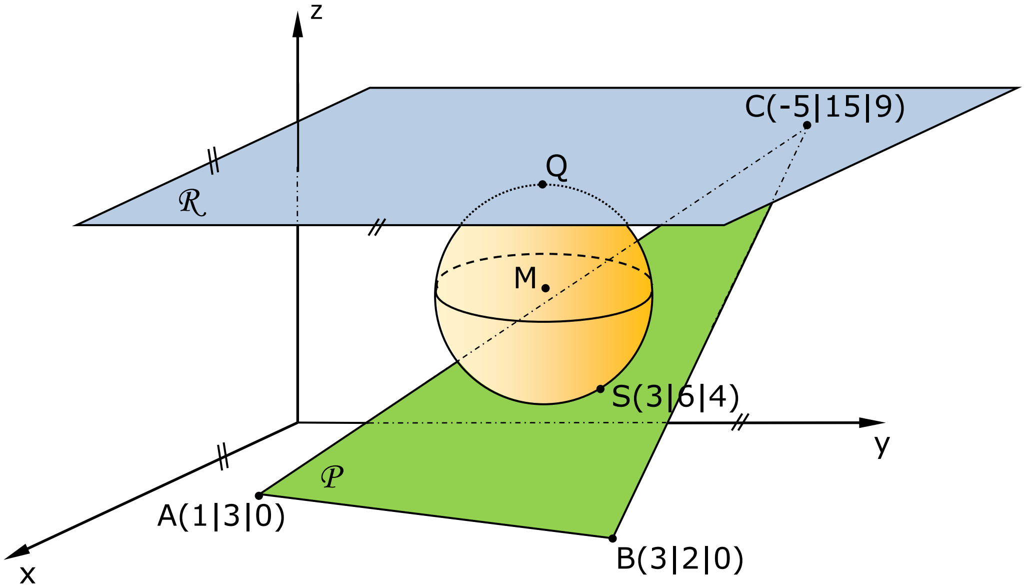


d. Prove that $F(x) = 4 \cdot e^{\frac{x-2}{2}} + c$ is an antiderivative of the function f.

e. The graph of f, together with the negative x-axis and the positive y-axis, enclose a region which stretches to infinity to the left. Calculate the area of this region by using and evaluating the antiderivative given in exercise d.

f. The graph of f, between x = 2 and x = b, with b > 2, rotates about the x-axis. The obtained solid of revolution has a volume of $V = 4\pi \cdot \left(e^3 - 1\right)$. Find the value of b.

Exercise 3 **Vector Geometry**	a	b	c	d_1	d_2	e_1	e_2	**Points**
	2	2	3	3	1	1	1	13



The diagram above is only a possible sketch of the following situation:

The points A, B and C define the plane $\mathcal{P}$. In addition, point C also lies in the plane $\mathcal{R}$: z – 9 = 0 which is parallel to the xy-plane.
Furthermore, a sphere with center M is located between both planes $\mathcal{P}$ and $\mathcal{R}$; the sphere touches the plane $\mathcal{P}$ in point S and the plane $\mathcal{R}$ in point Q.

a. Show that the plane $\mathcal{P}$ has the Cartesian equation $\mathcal{P}: x + 2y - 2z - 7 = 0$.

b. By what angle φ is plane $\mathcal{P}$ inclined *(= geneigt)* with respect to the xy-plane?

c. The straight line ℓ_{CS} through points C and S intersects the xy-plane at point T. Is T closer to point A or to point B? Justify your answer by a calculation.

d. The straight line ℓ_{AB} passes through the points A and B.

   d_1. How far is point C away from the line ℓ_{AB}?

   d_2. Determine the area of the triangle Δ_{ABC}.

e. As described above, the sphere with center M touches the planes $\mathcal{P}$ and $\mathcal{R}$.

   e_1. Find the x- and y-coordinates of the center M(x/y/6) of the sphere.

   e_2. Determine the coordinates of the point Q at which the sphere touches the plane $\mathcal{R}$.

Exercise 4 Probability	a_1	a_2	a_3	b_1	b_2	**Points**
	1	0.5	2	1	0.5	
	b_3	b_4	b_5	c		12
	1	1	3	2		

In a box there are 5 red, 3 white and 2 yellow balls. Balls of the same color are indistinguishable from each other.

a. A child places all balls one after the other on a table. How many different arrangements are there,

   a_1. if there are no further restrictions?

   a_2. if all balls of the same color should be next to each other?

   a_3. if two red balls are never allowed to lie next to each other?

b. In a first game, the balls are randomly drawn with replacement. What is the probability

   b_1. to draw exactly 2 yellow balls in 4 draws?

   b_2. not to draw a yellow ball in 10 draws?

   b_3. to draw at least 3 red balls in 5 draws?

   b_4. to draw the 7^{th} white ball in the 10^{th} draw?

   b_5. How many draws at least do you have to make to have at least one yellow ball with at least 99% probability?

c. In a second game, three balls are randomly selected simultaneously in one draw. For each white ball the player receives 2 Fr., for each different colored one he has to pay one franc. What average profit can the player expect per game?