KANTON LUZERN
Bildungs- und Kulturdepartement

Kantonsschule Alpenquai Luzern

Written Matura Exam 2017

Subject	*Mathematics Basic Course*
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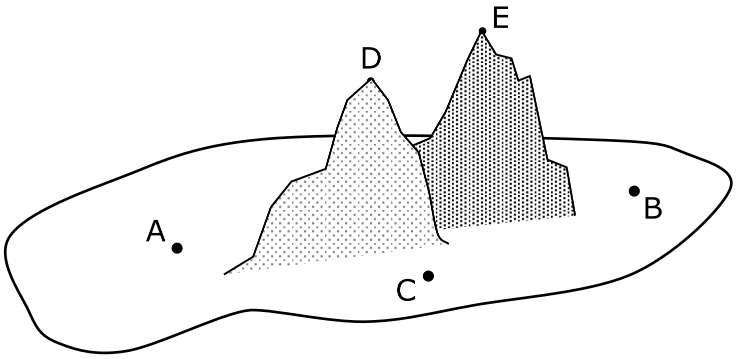
Classes	*6c / 6m*
Date of the exam	*Friday, 19th of May, 2017*
Time	*180 minutes*
Aids allowed	- *"Mathematics Formulary", Adrian Wetzel* - *A dictionary (book, no electronic translator)* - *TI-30, Voyage 200 without user manual*
Instructions	- *Importance is attached to a proper and clear representation.* - *Write each exercise on a separate sheet of paper.* - *All solutions must show the steps leading to the result.* - *Put your personal number, your name and your class on every sheet of paper.*
Maximum points per exercise	*Exercise 1: 13* *Exercise 2: 14* *Exercise 3: 8* *Exercise 4: 11* ***Total: 46*** *41 points are required for a grade of 6.*
Number of pages	*4 (including the title page)*

Exercise 1 Vector Geometry	a	b	c	d	e	f	g	Points
	1	1	2	1	2	3	3	13

The three villages $A(9|2|0)$, $B(1|8|0)$ and $C(9|12|0)$ are in the ground plane.
Two mountain peaks are located at the coordinates $D(5|4|6)$ und $E(3|6|8)$.



a. Determine the distance between the two mountain peaks *D* and *E*.

b. Show with a calculation that the village *A* and the mountain peaks *D* and *E* are not all located on one straight line.

c. A sunbeam hits via mountain peak *E* the village *A*.
What is the angle between the sunbeam and the ground plane *ABC*?

d. On which point *P* of the ground plane *ABC* does it look as if the peak *E* lies just behind peak *D*?

e. A helicopter flies in a straight line from peak *D* to peak *E*. Where (in which point) is it closest to village *B*?

f. A viewing tower with a height of 3 is planned on the straight road between the villages *B* and *C. T*, the top of the viewing tower shall be at the same distance from both mountain peaks. Calculate the coordinates of *T*, the top of the viewing tower and the coordinates of *F*, the base (foundation) point of the viewing tower.

g. An aerial cableway (=Seilbahn) connects village *A* with the mountain peak *D* along a straight line. An observer is located at the top of the viewing tower *T*. Where is the aerial cableway in the moment, when the observer sees the cableway cabin and village *C* under a right angle?

Exercise 2 Calculus	a	b	c	d	e	Points
	6	2.5	1	3	1.5	14

The functions $f(x) = \dfrac{36 - x^2}{x^2 + 9}$ and $g(x) = 5 \cdot e^{-x^2/9} - 1$ are given.

a. Determine domain, symmetry, zeros, maximum and minimum points (= extrema), points of inflection and asymptotes of f and then draw the graph of the function f. *Units: 2 squares or 1cm.*

b. Determine the area of the region between the graphs of f and g located above the x-axis.

c. The graph of g, the y-axis and the horizontal asymptote y = -1 of g enclose a region for $x \geq 0$ which stretches to infinity to the right. Calculate the finite area of this region.

d. The horizontal asymptote of f intersects the y-axis in point P. The line $\ell(x) = k$ with $-1 < k < 4$ intersects the graph of f in the points Q and R. Find the value of k in such a way that the area of the triangle PQR is as large as possible.

e. At point P(u/v) of the graph of f with 0 < u < 1, the function value v equals the value of the slope of the normal line at point P(u/v). Find the values of u and v.

Exercise 3 Calculus	a	b_1	b_2	Points
	3.5	3	1.5	8

The graph of a third order polynomial f passes through the origin, touches the positive x-axis and encloses with the positive x-axis an area of A = 8. Finally, the point $H\left(\frac{4}{3} / y_H\right)$ is a high point (= maximum) of the graph of f.

a. Determine the function equation of the polynomial f.

Solve the following exercises with the polynomial $f(x) = \dfrac{3}{8}x^3 - 3x^2 + 6x$.

b_1. The area enclosed by the graph of f, its tangent t at point $P(1/y_P)$ and the x-axis rotates about the x-axis. Calculate the volume of the resulting solid of revolution.

b_2. The graph of polynomial f, its tangent t at point $P(1/y_P)$ and the vertical line x = b with 1 < b < 6 enclose an area A_1. The area A_2 is enclosed by the graph of the polynomial f and the x-axis. Find the value of b so that the area A_1 is twice as big as A_2.

<table>
<tr><td rowspan="4">Exercise 4
Probability</td><td>a</td><td>b</td><td>c</td><td>d_1</td><td>d_2</td><td rowspan="2">Points</td></tr>
<tr><td>0.5</td><td>1</td><td>1</td><td>1</td><td>1</td></tr>
<tr><td>d_3</td><td>d_4</td><td>e</td><td>f</td><td>g</td><td rowspan="2">11</td></tr>
<tr><td>1</td><td>1</td><td>2</td><td>1</td><td>1.5</td></tr>
</table>

The First Nations (=Ureinwohner) of Vancouver Island (Canada) hold their annual meeting called potlach in Victoria. There are delegations of 14 Nations (=Stämme). Each Nation resides in a tipi (tent). All the tipis are set up along a straight line. The place of each Nation is drawn by lots.

a. How many different arrangements of the Nations are possible?

b. Determine the probability that the tipis of the two friends Mel from the Malahat Nation and Sue of the Sooke Nation are assigned neighboring places.

c. This year, six Nations have brought a blue tipi, the remaining eight reside in red tipis. How many different arrangements according to colors are there?

On seven days, games are part of the celebrations. Each day, a member of each Nation is determined by lot to represent the Nation at the game of that day. The delegation of the Malahat Nation consists of eight people, Mel is one of them.
Determine the probability that Mel

d_1. … may never represent her Nation.

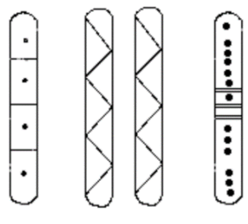
d_2. … represents her Nation on the third day for the first time.

d_3. … may represent her Nation exactly twice in these seven days.

d_4. … may represent her Nation at least three times in these seven days.

e. The delegation of the Sooke Nation consists of eleven members, among them are four members of Sue’s family. How many nights does Sue’s cousin Andrew, who is not a member of the delegation, have to book a hotel at least so that he sees a member of Sue’s family compete at least once with a probability of at least 90%?

Four sticks are thrown at the traditional evening game. The front of these sticks is carved, as shown on the right-hand side, the back is blank. The sticks fall at random on one side (carved or blank), the order of the sticks does not matter.



f. Calculate the probability that at one throw of the four sticks at least one of the zig-zag sides (like in the middle of the drawing) is visible.

Points are assigned at each throw. If all four blank sides are visible, the player gets zero points. Is one carved side visible, the player receives one point, and for each further carved side that is visible, the number of points is doubled. If for instance three carved sides are visible, the player gets four points.

g. How many points are reached on average with one throw?