KANTON LUZERN
Bildungs- und Kulturdepartement

Kantonsschule Alpenquai Luzern

Mathematics Basic Course
Written Matura Exam 2015

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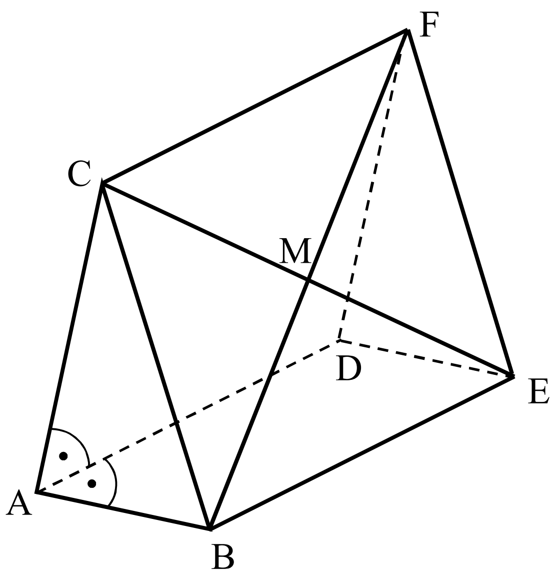
Classes	*6Lc / 6Ld / 6Ra/b*
Date of the exam	*Tuesday, May 26th, 2015*
Duration	*180 minutes*
Aids allowed	*Adrian Wetzel "Mathematics formulary"* *Calculator TI30, Voyage 200, no reference book* *A dictionary (book, no electronic translator)*
Instructions	*- Importance is attached to a proper and clear representation.* *- Write each exercise on a separate sheet of paper.* *- All solutions must show the steps leading to the result.* *- Put your personal number, your name and your class on every sheet of paper.*
Maximum points per exercise	*Exercise 1: 10.5* *Exercise 2: 11.0* *Exercise 3: 11.5* *Exercise 4: 10.0* ***Total: 43.0***
Points required for a grade of 6	*38 points*
Number of pages	*5 (title page included)*

Exercise 1 - Vector Geometry	a	b	c	d	e	**Points**
	1.5	2.5	1	2.5	3	**10.5**

The triangular, right prism ABCDEF (*see sketch below*) is given by the vertices $A(33/18/40)$, $E(0/12/0)$, $F(-7/-12/15)$ and $M\left(\frac{17}{2}/9/\frac{55}{2}\right)$ where M is the midpoint of the quadrilateral CBEF.



a. Calculate the coordinates of the vertices B and C and verify that D has the coordinates $D(9/0/0)$.

b. Show that the triangle DEF has a right angle at the vertex D, and determine the volume of the prism ABCDEF.

c. Set up the Cartesian equation of the plane $\mathcal{P}_{AEM}$.

d. The line ℓ_1 passes through the centroid S of the triangle DEF and through the point M. Calculate the intersection angle φ between the line ℓ_1 and the xz-plane.

e. Calculate the minimum distance of the vertex D from the line ℓ_2 through the vertices C and E.

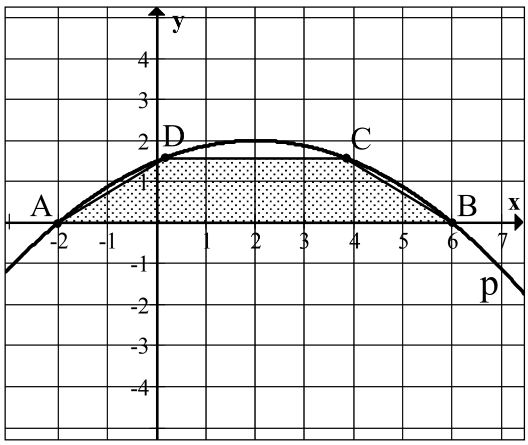
Exercise 2 – Calculus	a	b	c_1	c_2	**Points**
	5.5	2	1.5	2	**11**

The functions $f(x)=\dfrac{x^3+x^2+4}{2x^2}$ with $x \neq 0$ and $p(x)=-\dfrac{1}{8}x^2+\dfrac{1}{2}x+\dfrac{3}{2}$ are given.

a. Determine the first three derivatives, domain, symmetry, zeros, maximum and minimum points (= extrema), points of inflection and asymptotes of f(x) and then draw the graph of the function f(x). *Units: 2 squares or 1cm.*

b. Prove that the parabola p(x) touches the curve of f(x). Find the coordinates of the points of contact S as well.

c. An isosceles trapezoid ABCD is inscribed into the region between the parabola p(x) and the x-axis. The points A and B are the intersection points of p(x) with the x-axis; C and D are points on the graph of p(x) (*see figure*).



c_1. Show that the area G of the trapezoid ABCD is given by the function

$$G(u)=p(u)\cdot(u+2)$$

where u is the x-coordinate of the point C(u/v).

c_2. Determine the coordinates of point C so that the inscribed trapezoid ABCD has the largest possible area.

Exercise 3 – Calculus	a	b	c	d	e	Points
	3	3	1	2	2.5	**11.5**

The function $f(x) = (x-2)\cdot e^{3-\frac{x}{2}}$ is given.

a. The curve f(x), its tangent of inflection and the x-axis enclose a region which stretches to infinity to the right. Calculate the area of this region.

b. An open upwards parabola $p(x) = ax^2 + bx + c$ intersects the curve f(x) at its zero and at x = 6; the area enclosed by the parabola p(x) and the curve f(x) measures A = $4\cdot e^2$. Determine the function equation of p(x).

c. The area enclosed by the curve f(x), the x-axis and the vertical line x = d (d > 2) rotates about the x-axis. Find the value of d if the volume of the corresponding solid of revolution measures V = $2\pi\cdot\left(e^4-13\right)$.

Now, we will consider the more general function $g(x) = (x-k)\cdot e^{3-\frac{x}{k}}$ depending on the parameter k with k > 0.

d. Find the value of k so that the maximum point H of g(x) has a y-coordinate of y = 3.

e. Prove: The intersection angle α of the graph of g(x) with the x-axis does not depend on the value of the parameter k. Calculate the angle α as well.

Exercise 4 – Combinatorics / Probability	a_1	a_2	b	c_1	**Points**
	0.5	1	1.5	1	
	c_2	c_3	d	e	**10**
	1	1	2	2	

Mrs. Molar is redecorating the waiting room (three walls and a window front) of her dental hygiene practice. There are seven different colors to choose from.

a. In how many different ways can she paint the walls of the waiting room if

a_1. all three walls should be painted in a different color;

a_2. exactly two of the three walls should be painted in the same color?

On each one of the freshly painted walls she identifies two places where she could present a diploma.

b. In how many different ways can she hang up her four diplomas if the only restriction is that there is at least one diploma on each wall? *Note that it matters on which one of the two places on a wall each diploma is positioned.*

Mrs. Molar knows out of experience that 60% of all her clients are women and 40% are men. There are seven half-hour sessions in the morning. These sessions are always booked out.

c. Calculate the probability that on a given morning

c_1. all seven sessions are booked for clients of the same sex;

c_2. there are exactly two female and five male clients booked;

c_3. there are more men than women booked.

d. There are always some clients that do not show up on the appointed date: On average, 7% of all men forget the appointment, whereas 4% of all women miss an appointment. What is the probability that all of 7 randomly selected clients will see the doctor as arranged?

e. A marketing company is hired to assess client satisfaction. The company telephones people from the customer list at random. Mrs. Molar is especially interested in knowing more about the reasons why clients forget about their appointment.

How many telephone numbers of clients does the marketing company need to have at least, if they want to reach at least one client who has forgotten the last appointment with a probability of at least 99 %? *Use 0.948 for the probability that a client doesn't forget an appointment, if you could not find this value.*