

Time: 180 minutes

All solutions must show the steps leading to the result. Importance is attached to a proper and clear representation. Each exercise is labelled with the maximum points. 45 out of 50 points are required for a mark of 6.

Aids allowed: Formula Book “Mathematical Formulas for Economists”, Springer
Mathematical Handbook of Formulas (additional material)
A dictionary (book, no electronic translator)
TI-30, TI-92, TI-92 plus, Voyage 200, without the user manual

The use of the aids is to be declared clearly.

✎ **Write each exercise on a new sheet of paper!**

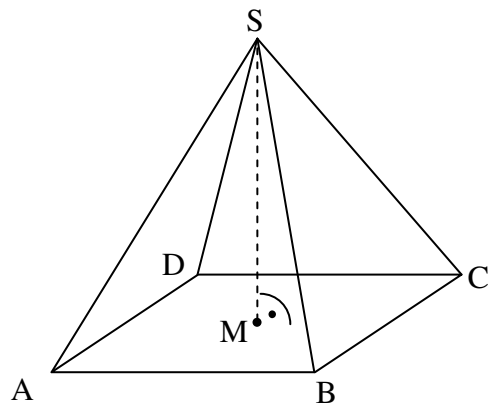
✎ **Write your personal number, your name and your class on every sheet of paper!**

Exercise 1

12 points

The points $A(0/0/0)$, $B(4/2/4)$ and $C(6/6/0)$ of a right regular pyramid $ABCDS$ with the square $ABCD$ as base and S as apex (=Spitze) is given (see figure).

- Determine a Cartesian equation of the plane \mathcal{P} passing through the points A , B and C .
- Find the coordinates of point D .
- The volume V of the pyramid $ABCDS$ is $V = 36$ volume units. Calculate the coordinates of point S with the z -coordinate of S being positive.



If you could not solve exercise c., take point $S(-1/7/2)$ to solve the following exercises.

- By how many degrees is the height MS of the pyramid tilted (=geneigt) with respect to the xy -plane?
- Which point Q on the edge AS is closest to point M ? Find the coordinates of Q .

Exercise 2

12 points

Let $f(x)$ be a 3rd order polynomial. The graph of $f(x)$ has a tangent with slope $\frac{9}{4}$ at point $P(0/3)$ and an inflection point at $Q(4/4)$.

- a. Find the function equation of $f(x)$.

If you could not solve exercise a., continue with $f(x) = \frac{4}{27}x^3 - \frac{4}{3}x^2 + 3x + 3$.

- b. First, determine the zeros and the stationary points (extrema) of $f(x)$, whereby you are not required to check the stationary points (extrema) with the second derivative. Second, sketch the graph of $f(x)$ for $-2 \leq x \leq 8$. **Units** in the coordinate system: One unit is two squares!
- c. The area enclosed by the graph of $f(x)$ and the tangent at the (local) maximum is rotated about the x -axis. Calculate the volume of this solid of revolution.
- d. The graph of $f(x)$, the x -axis and the two parallel lines $x = k$ and $x = k + 4$ with $k \geq 0$ enclose an area. Find the value of k for which the area is a minimum and draw this minimum area in the sketch of exercise b.

Exercise 3

11 points

The functions $f(x) = \frac{x^2 - 2x + 2}{x - 1}$ and $g(x) = -x^2 + 7x - \frac{23}{4}$ are given.

- a. Determine the asymptotes of the function $f(x)$.
- b. Calculate the area enclosed by the curves of the two functions in the first quadrant.
- c. The tangent to the curve of $g(x)$ at point P intersects the line $y = x + 2$ perpendicularly. Find the coordinates of point P .
- d. A triangle ABC is inscribed into the area described in exercise b. Point A is the left intersection point of the two curves in the first quadrant, and the side BC is parallel to the y -axis. Determine the x -coordinate of B and C in such a way that the area of the triangle ABC is a maximum.

Exercise 4

10 points

An urn at a fair (=Jahrmarkt) contains one red and four black balls. At each draw, one ball is randomly selected, its color is recorded and then the ball is put back into the urn.

- a. There are six draws. Calculate the probability that
 - i. a black ball is drawn six times;
 - ii. the ball which is drawn second is red;
 - iii. there are at least four black balls drawn.

- b. How many draws must be done at least for the probability to be at least 95% that at least one red ball is drawn?

- c. At the price of one Swiss Franc, the following game is offered: If the participant manages to draw at least two red balls in three draws, he receives nine Swiss Francs; otherwise he receives nothing and in addition loses his stake (=Einsatz) of one Swiss Franc. Find the expected gain or loss at this game.

Exercise 5

5 points

In a box at a book bazar there are 20 different books, 12 novels and 8 non-fiction books, on sale.

- a. In how many different ways can a customer take a selection of books if he would like to have at least two novels and at least one non-fiction book, but a total of four books at most?

- b. The 20 books are lined up next to each other on a bookshelf.
 - i. In how many different ways can this be done?
 - ii. Find the number of different arrangements, if the line-up from left to right should be as follows: first 9 novels, then the 8 non-fiction books and finally the remaining 3 novels.