KANTON LUZERN Kantonsschule Luzern

Obergymnasium

MATURA 2008
Mathematics
Basic Course

Class 6Lc (obr)

Time: 180 minutes

All solutions must show the steps leading to the result. Importance is attached to a proper and clear representation. Each exercise is labelled with the maximum points. 41 out of 49 points are required for a mark of 6.

Aids allowed: Formula Book “Mathematical Formulas for Economists”, Springer
Mathematical Handbook of Formulas (additional material)
A dictionary (book, no electronic translator)
TI-30, TI-92, TI-92 plus, Voyage 200, without the user manual
The use of the aids is to be declared clearly.

☞ **Write each exercise on a new sheet of paper!**
☞ **Write your personal number, your name and your class on every sheet of paper!**

Exercise 1 **13 points**

Given are a cube with sidelength 6 [units] and the point A(6/0/3) as shown in the sketch below. The trapezoid ABCD is the intersection of plane $\mathcal{P}$ and the cube. The coordinates of all the other points can be taken from the sketch.

Determine

a. a Cartesian equation of the plane $\mathcal{P}$.

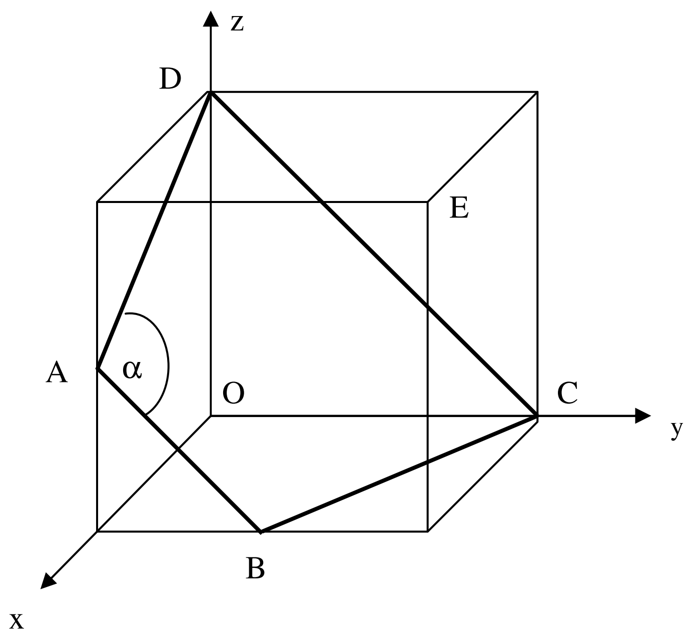
b. the intersection point S of the cube-diagonal OE with the plane $\mathcal{P}$.

c. the angle $\alpha = \sphericalangle(DAB)$ in the trapezoid ABCD.

d. the distance d between point E and plane $\mathcal{P}$.

e. Prove that the trapezoid ABCD is isosceles.

f. Calculate the area of the trapezoid ABCD.



Exercise 2 — 14 points

The function $f(x) = e^{-x}\left(x^2 - x - 2\right)$ is given.

a. Determine the zeros, the stationary points, the inflection points, the symmetries and the asymptotes of f(x) and sketch its graph.

b. Determine the area of the region that stretches to infinity to the right and is enclosed by the graph of f(x) and the x-axis.

c. The bounded (= beschränkt) area enclosed by the graph of f(x) and the x-axis is rotated about the x-axis. Calculate the volume of the solid of revolution.

d. The graph of a third order polynomial g(x) touches the graph of f(x) at point P(0/-2). The tangent at the minimum point of g(x) intersects the graph of g(x) in a second point P(-1/-4). Find the function equation of g(x).

Exercise 3 — 12 points

The functions $f_k(x) = -\frac{1}{9k}x^3 + kx$ and $g_k(x) = \frac{k}{9}x^3 - \frac{1}{k}x$ with $k > 0$ are given.

a. Let k = 3. Determine the equation of the inflection tangent at the graph of function $g_3(x)$.

b. Calculate the intersection angle φ between the graphs of $f_k(x)$ and $g_k(x)$ at the origin.

c. The sides of a rectangle are defined by the coordinate-axes and two lines that run parallel to the coordinate-axes and contain the maximum point of the graph of $f_k(x)$. This rectangle is divided into two parts by the graph of $f_k(x)$. Calculate the areas of these two parts and determine the proportion of their areas.

d. The graphs of $f_k(x)$ and $g_k(x)$ enclose an area for $x \geq 0$. Find k in order for this enclosed area to be a minimum, and calculate that minimum area.

Exercise 4 **10 points**

From previous experience, 25% of all customers of a travel agency book a boat trip to Denmark.

a. Find the probability that there are exactly 50 bookings for the boat trip to Denmark out of 200 bookings.

b. How many bookings must be made at least so that the probability that there is at least one booking for the boat trip to Denmark is higher than 95%?

c. On the boat to Denmark, a travel agency displays brochures for a bus tour. From previous experience it is known that 65% of the passengers read the brochure. 30% of the readers book the bus tour spontaneously, the rest of the readers will book the bus tour with a probability of 40% later. Find the probability that a randomly selected passenger on the boat to Denmark will book the bus tour.

d. Passengers of age (= volljährig) whose birthday is on the day of the boat trip to Denmark are invited by the captain to a glass of Champagne. If there are 1000 passengers of age, on how many days can the captain expect to have

   i. no guests,

   ii. more than four guests?

   Assumptions:
   A year has 365 days. Each day of a year is equally likely to be a birthday.