Kantonsschule Alpenquai Luzern

Written Matura Exam 2023

Subject	Mathematics Basic Course
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Classes	G19d / G19l / G19m
Date of the exam	May 26, 2023
Time	3 hours
Aids allowed	• "Mathematics Formulary", Adrian Wetzel • A dictionary (book, no electronic translator) • TI-30X Pro (no handbook)
Instructions	• Importance is attached to a proper and clear representation. • Write each exercise on a separate sheet of paper. • All solutions must show the steps leading to the result. • Put your personal number, your name and your class on every sheet of paper. • The use of the aids must be clearly stated.
Maximum points per exercise	Exercise 1: 11 Exercise 2: 12 Exercise 3: 4 Exercise 4: 7 Exercise 5: 11 Total: 45 40 points are required for a grade of 6, and 23 points for a grade of 4.
Number of pages	5 (including title page)

	a	b	c	d	e	
Exercise 1: Vector geometry	2.5	1.5	1	3.5	2.5	**11 points**

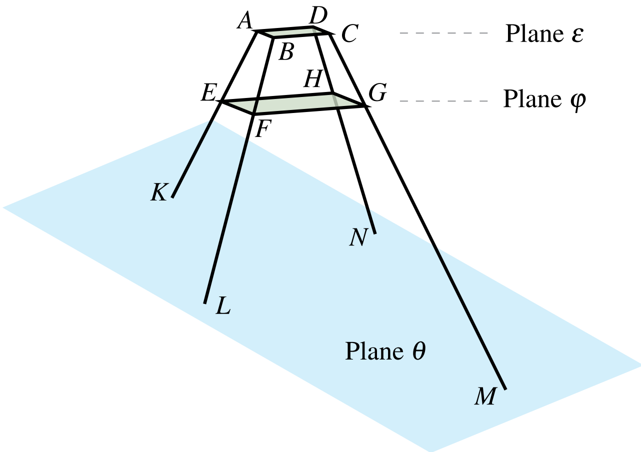
Given are the points:

$A(5|12|40)$, $B(13|8|40)$, $D(11|24|40)$,
$E(-2|8|24)$, $G(26|24|24)$, $H(10|32|24)$,

as well as the plane:

$$\theta : 4x + y + 8z + 294 = 0$$

A viewing tower stands on a slanted plane θ and consists of two parallel platforms: Platform $ABCD$ in the plane ε and Platform $EFGH$ in the plane φ. The tower is supported by the poles AK, BL, CM and DN.



a) If the poles AK, BL, CM and DN are prolonged, they intersect in a point S. Find the coordinates of this point S.

b) Show, that $\angle EHG = 90°$.

c) The points $EFGH$ form a rectangle. Determine the coordinates of point F.

d) The points A, B, C, and D are in the plane ε.
   i) Find the Cartesian equation of the plane ε.
   ii) Calculate the angle between the planes ε and θ.

e) The tower should be secured by an iron cable running from point E to the plane θ. Determine the point P on the plane θ so, that the cable has minimal length. Calculate as well the length of that cable.

	a	b	c	d	
Exercise 2: Calculus	1	7.5	2	1.5	**12 Punkte**

The function $f(x) = x^4 - 2x^2 + 1$ is discussed.

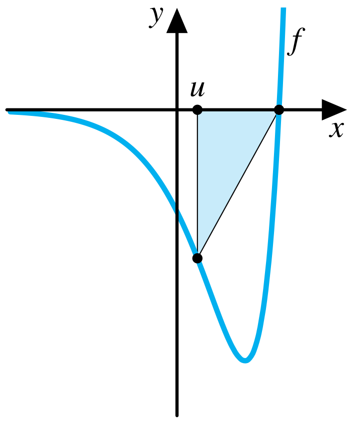
a) Show that the function equation of f can be written as
$f(x) = (x+1)^2 \cdot (x-1)^2$.

b) Draw a coordinate system at which 4 squares correspond to one unit. All the points that are calculated in this subtask should be drawn into this coordinate system.
   i) Determine all points of intersection of f with the coordinate axes.
   ii) Show that the extrema are located at $x = -1$, $x = 0$ and $x = 1$.
   iii) Determine the coordinates of the extrema, the inflection points of f and draw the graph of f.

c) The parabola p contains all three extrema of f. The vertex of p coincides with the middle extremum of f.
Determine the equation of p and draw the graph of p into the coordinate system.

d) Determine the size of the area F, that is enclosed by the graphs of the two functions in the first quadrant.

Exercise 3: Calculus	**4 points**

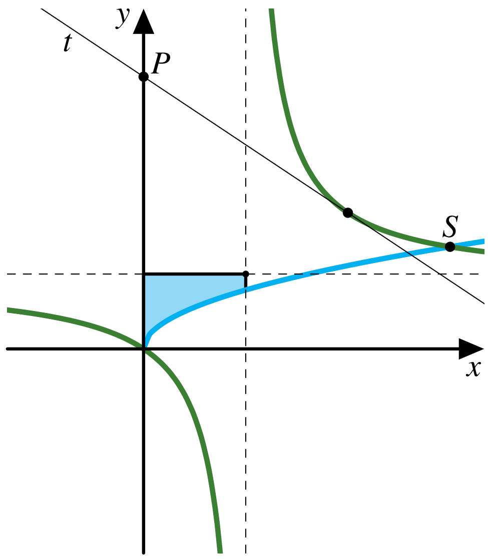


The graph of $f(x) = (x-3) \cdot e^x$ has exactly one zero.
This zero forms with the points $(u|0)$ and $(u|f(u))$ a triangle below the x-axis.
For which value of u is the area of this triangle maximal?

	a	b	c	
Exercise 4: Calculus	2.5	2	2.5	**7 points**

Shown at the right-hand side are the graphs of the functions

$f(x) = \sqrt{x}$ and $g(x) = \dfrac{2x}{x-3}$



a) Determine algebraically the coordinates of the point of intersection S of the graphs f and g.

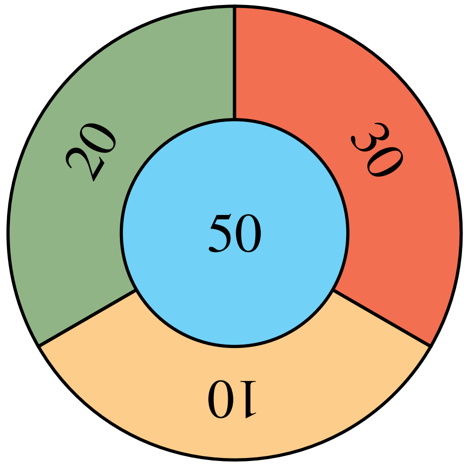
b) Show that the tangent t to the graph of g at $x = 6$ intersects the y-axis in point $P(0|8)$.

c) The graph of g has a horizontal and a vertical asymptote, respectively. The area that is enclosed by these asymptotes, the y-axis and the graph of f, is rotated about the x-axis. Determine the volume of the resulting solid of revolution.

	a	b	c	d	e	f	
Exercise 5: Probability	1.5	2	2	2	1	2.5	**11 points**

Anouk and Ben are having a darts duel. They shoot with darts at the target shown.

Anouk hits the blue sector (50 points) with a probability of 55%, the other sectors (10, 20 and 30 points) are hit with a probability of 15% each. Ben, on the other hand, hits the blue sector with a probability of 49%, the other sectors are hit with a probability of 17% each. For all subtasks, we assume that Anouk and Ben hit the target and thus one of the four colored sectors with each dart.



a) Ben throws five darts. How many outcomes are possible,
   i) if the five darts can be distinguished?
      (e.g. red | red | yellow | blue | green ≠ green | red | blue | red | yellow)
   ii) if the five darts can be distinguished and the outcome is: twice the yellow, twice the green and once the blue sector have been hit?
      (e.g. green | blue | yellow | green | yellow ≠ yellow | green | blue | green | yellow)
   iii) if the five darts can not be distinguished?
      (e.g. red | red | yellow | blue | green = green | blue | red | yellow | red)

b) Determine the probability that Anouk scores 100 or 110 points with three darts.

c) Anouk throws 20 darts. Calculate the probability that
   i) she hits the blue sector exactly 8 times.
   ii) she hits the blue sector at most 7 times.

d) With a probability of more than 99%, Anouk wants to hit the blue section with at least one dart. Calculate how many darts Anouk has to shoot at least to achieve this.

e) What average score can Anouk expect with one throw of a dart?

f) Anouk and Ben play the following game multiple times. They throw one dart each. The one that hits the sector with more points wins. The winner receives the points thrown (e.g. if Ben wins with a dart in the red sector, he gets 30 points). The loser gets no points. If both throw in the same sector, they get no points. How many points can Ben expect on average per game?

Kurzlösungen

Aufgabe 1: Vektorgeometrie

a) $S(12|16|56)$

b) $\overrightarrow{HE} \cdot \overrightarrow{HG} = \begin{pmatrix} -12 \\ -24 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 16 \\ -8 \\ 0 \end{pmatrix} = 0$

c) $F(14|0|24)$

d) i) $\varepsilon : z = 40$

ii) $\alpha = 27.27°$

e) $P(-26|2|-24)$

$d = \left|\overrightarrow{PE}\right| = 54$

Aufgabe 2: Analysis

a) $f(x) = (x+1)^2 \cdot (x-1)^2 = (x^2-1)^2 = x^4 - 2x^2 + 1$

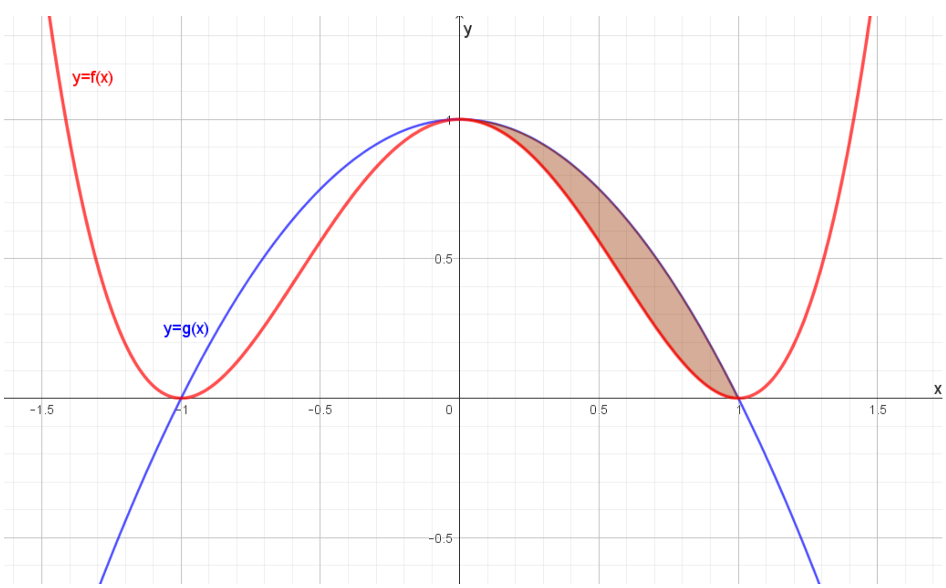
b) Nullstellen: $N_1(-1|0)$, $N_2(1|0)$

y-Achsenabschnitt: $B(0|1)$

Extrema: $H(0|1)$, $T_1(-1|0)$, $T_2(1|0)$

Wendepunkte: $W_1(-0.58|0.44)$, $W_2(0.58|0.44)$

Graph:



c) $p(x) = -x^2 + 1$

d) $F = \dfrac{2}{15}$

Aufgabe 3: Analysis

Fläche: $A(u) = -\frac{1}{2} \cdot \left(u^2 - 6u + 9\right) \cdot e^u$

Die Fläche des Dreiecks wird für $u = 1$ maximal.

Aufgabe 4: Analysis

a) Schnittpunkt: $S(9|3)$

b) $t(x) = -\frac{2}{3}x + 8$

c) vertikale Asympotote: $x = 3$

horizontale Asympotote: $y = 2$

Volumen: $\pi \int_0^3 2^2\,dx - \pi \int_0^3 \left(\sqrt{x}\right)^2 dx = \frac{15}{2}\pi \approx 23.56$

Aufgabe 5: Wahrscheinlichkeit

a) i) $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 4^5 = 1024$

ii) $\frac{5!}{2! \cdot 2! \cdot 1!} = 30$

iii) $\binom{5+4+1}{5} = \binom{8}{5} = 56$

b) $100 = 20 + 30 + 50 \quad \leftarrow \quad 3! = 6$ Möglichkeiten

$110 = 10 + 50 + 50 \quad \leftarrow \quad \frac{3!}{2} = 3$ Möglichkeiten

$110 = 50 + 30 + 30 \quad \leftarrow \quad \frac{3!}{2} = 3$ Möglichkeiten

$P(100 \text{ Punkte oder } 110 \text{ Punkte}) = 0.2475$

c) i) $P(X = 8) = \binom{20}{8} \cdot 0.55^8 \cdot 0.45^{12} = 0.073$

ii) $P(X \leq 7) = \sum_{i=0}^{7} \binom{20}{i} \cdot 0.55^i \cdot 0.45^{20-i} = 0.058$

d) Anouk muss mindestens 6 Mal einen Pfeil schiessen.

e) $E(X) = 36.5$ Punkte

f) $E(\text{Punktegewinn von Ben}) = 0 \cdot 10 + 0.0255 \cdot 20 + 0.051 \cdot 30 + 0.2205 \cdot 50 = 13.065$ Punkte