KANTON LUZERN
Bildungs- und Kulturdepartement

Kantonsschule Alpenquai Luzern

Written Matura Exam 2020

Subject	Mathematics Basic Course
Teacher	Roman Oberholzer roman.oberholzer@edulu.ch
Class	6k
Date of the exam	Friday, 15th of May, 2020
Time	180 minutes
Aids allowed	- "Mathematics Formulary", Adrian Wetzel - A dictionary (book, no electronic translator) - TI-30X Pro Multiview
Instructions	- Importance is attached to a proper and clear representation. - Write each exercise on a separate sheet of paper. - All solutions must show the steps leading to the result. - Put your personal number, your name and your class on every sheet of paper.
Maximum points per exercise	Exercise 1: 11 Exercise 2: 12 Exercise 3: 12 Exercise 4: 10 **Total: 45** 38 points are required for a grade of 6.
Number of pages	5 (including title page)

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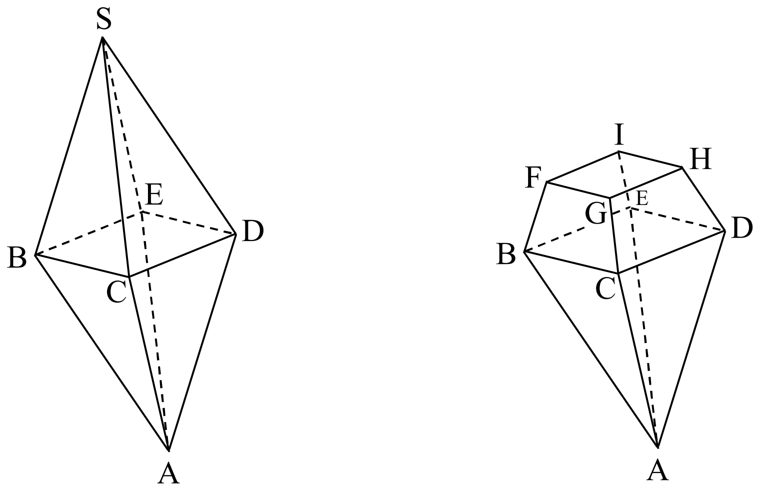
Exercise 1 Vector Geometry	a	b	c	d	e	Points
	3	1.5	1.5	2.5	2.5	11

Diamonds could not be processed for a long time. In the 14th century, the facets (= *Oberflächen des Diamanten*) could be polished for the first time and the diamonds were formed as a octahedron *(= Achtflächner)* ABCDES as shown at the left.

From the 15th century onwards, it was possible to create a so-called tablet FGHI, a flat slab *(= Platte)*, by grinding off *(= abschleifen)* the apex *(= Spitze)* S. The solid shown below at the right is thus composed of the straight *(= gerade)* square pyramid ABCDE and the frustum *(= Pyramidenstumpf)* BCDEFGHI.



The points A(-4/11/7), B(-1/2/4), C(1/6/0), D(5/8/4), E(3/4/8) and F(2/1/3) are given.

a. Calculate the Cartesian equation of the plane $\mathcal{P}$, passing through the points B, C and D, and show that point E lies on this plane as well.

b. Prove that the quadrilateral BCDE is a square.

c. Originally, both pyramids ABCDE and BCDES were straight square pyramids, being symmetrical to each other. Determine the coordinates of the apex S.

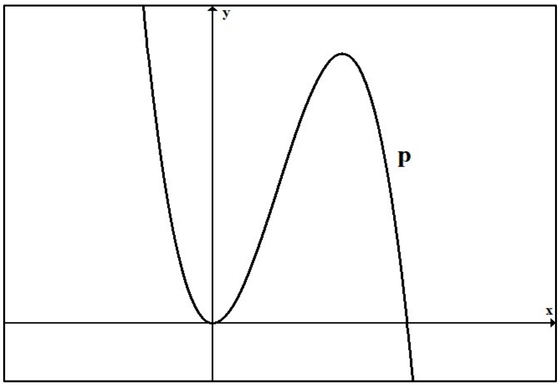
Continue with the apex S(8/-1/1).

d. The tablet FGHI is parallel to the square BCDE. Determine the coordinates of the point H.

e. By how many percent is the volume of the reduced diamond ABCDEFGHI *(shown above at the right)* smaller than the volume of the original octahedron ABCDES *(shown above at the left)*?

Exercise 2	a	b	c	d	Points
Calculus	3.5	1.5	3.5	3.5	12

a. The graph of a polynomial p of third order touches the x-axis at the origin and intersects the x-axis at x = 3. The area under the graph of p between these two zeros measures A = 13.5. Determine the function equation of the polynomial.

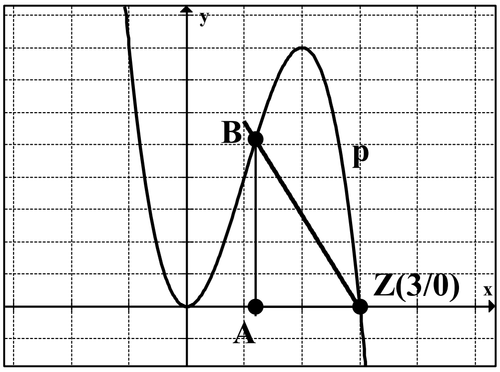


Continue in the following exercises with $p(x) = -2x^3 + 6x^2$.

b. The point Q lies on the graph of p in the second quadrant. The graph of p has in Q the same slope as in its zero Z(3/0). Calculate the coordinates of point Q.

c. The points A(u/0), Z(3/0) and B(u/p(u)) form a triangle AZB in the first quadrant.

Determine the coordinates of the point B in such a way that the triangle AZB has an area as big as possible. *The check of the maximum is required.*



d. Now we will consider the general polynomial $\overline{p}(x) = ax^3 + bx^2$. The points $\overline{Q}$ and $\overline{Z}$ (where $\overline{Q} \neq \overline{Z}$) lie on the graph of $\overline{p}$ where $\overline{Z}$ is the zero of the graph of $\overline{p}$, different from the origin. The graph of $\overline{p}$ has in $\overline{Q}$ and $\overline{Z}$ the same slope. Express the x-coordinate of $\overline{Q}$ in terms of a and b.

Exercise 3	a	b	c	d	e	**Points**
Calculus	5	2	2	1.5	1.5	12

The function $f(x)=\dfrac{3x^2-12x}{(x-1)^2}$ with the derivatives

$$f'(x)=\frac{6(x+2)}{(x-1)^3} \qquad f''(x)=\frac{-6(2x+7)}{(x-1)^4} \qquad \text{and} \qquad f'''(x)=\frac{36(x+5)}{(x-1)^5}$$

is given.

a. Determine the domain, zeros, stationary points (maximum and minimum points), points of inflection and asymptotes of f and then draw the graph of the function f. *Units: 2 squares or 1cm.*

b. Show that the function $F(x)=-6\cdot\ln|x-1|+\dfrac{9}{x-1}+3x+c$ is an antiderivative of the function f.

c. The graph of f, its horizontal asymptote and the vertical line x = 4 enclose a region which stretches to infinity to the right. Examine *(= untersuchen)* if this region has a finite size or not. Justify your answer by a calculation.

d. The line t is the tangent to the graph of f at x = 7. Show that t passes through the origin.

e. The area between the tangent t from exercise d., the graph of f and the x-axis enclose an area for $x \geq 0$ that rotates about the x-axis. Use your calculator to find this volume of revolution.

Exercise 4 Probability	a_1	a_2	b_1	b_2	c	**Points**
	1	0.5	0.5	0.5	2	
	d_1	d_2	e_1	e_2		10
	1	1.5	1	2		

A role-playing game is a game in which the six players assume the roles of characters in a fictional setting.

a. The game master has brought six playing cards with one hero character on each card.

a_1. Two cards show the identical pixie *(= Elfe)*, three cards show the identical dwarf and one card shows a huntress. In how many different ways can the hero characters be distributed among the six players?

a_2. The six players come to the game master one after the other. How many different orders *(= Reihenfolge)* of the players are there?

Furthermore, the game master distributes three identical elixirs *(= Heiltrank)*.

b. How many different distributions of the elixirs among the six players are there if

b_1. no player can get more than one elixir;

b_2. one player gets exactly two elixirs?

In the fictional setting of the game, the heroes must shoot with bow and arrow *(= mit Pfeil und Bogen schiessen)*. The probability that a specific hero hits the target, is determined each time by rolling a die with 20 faces, numbered from 1 to 20. The huntress hits the target if the die shows a number equal to or smaller than 13.

c. How many times does the huntress have to shoot at the target at least so that she hits it at least once with a probability of at least 99.9%?

d. For testing purposes, the huntress may shoot at the target 5 times. Calculate the probability that she

d_1. never hits the target;

d_2. hits the target at least 3 times.

e. After the test phase, the shooting starts in earnest: Every shot counts, and for every hitting of the target, the huntress gets a silver coin. She may shoot until she misses for the first time, but she can shoot a maximum of 5 times.

e_1. Find the probability that the huntress wins exactly 3 silver coins.

e_2. How many silver coins can the huntress expect to win in this game?

Short Answers

Exercise 1 [Vector Geometry]

a) $\mathcal{P}$. 2x – 2y – z + 10 = 0 insert point E → equation correct

b) $\overrightarrow{BC} = \begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix} = \overrightarrow{ED}$ and $\overrightarrow{CD} = \begin{pmatrix} 4 \\ 2 \\ 4 \end{pmatrix} = \overrightarrow{BE}$ with $\left|\overrightarrow{BC}\right| = \left|\overrightarrow{CD}\right| = 6$ → BCDE is a rhombus

$\overrightarrow{BC} \bullet \overrightarrow{CD} = 0 \rightarrow \overrightarrow{BC} \perp \overrightarrow{CD}$ → BCDE is a square

c) S(8/-1/1)

d) H(6/5/3)

e) volume of octahedron = 216 volume of the piece grinded-off = 32

loss of volume = 14.81%

Exercise 2 [Calculus]

a) $p(x) = -2x^3 + 6x^2$ b) Q_1(-1/8), Q_2(3/0)

c) target function: $A(u) = \frac{1}{2} \cdot (3-u) \cdot p(u) = u^4 - 6u^3 + 9u^2$ has a maximum for $u = \frac{3}{2}$

$\rightarrow B\left(\frac{3}{2} / \frac{27}{4}\right)$

d) zeros: x = 0 or $x = -\frac{b}{a}$ $x_{\overline{Q}} = \frac{b}{3a}$

Exercise 3 [Calculus]

a) domain $\underline{ID = \mathbb{R} \setminus \{1\}} = \{x \in \mathbb{R} / x \neq 1\}$

symmetry no symmetry (mixed exponents)

zeros zero Z_1(0/0), Z_2(4/0)

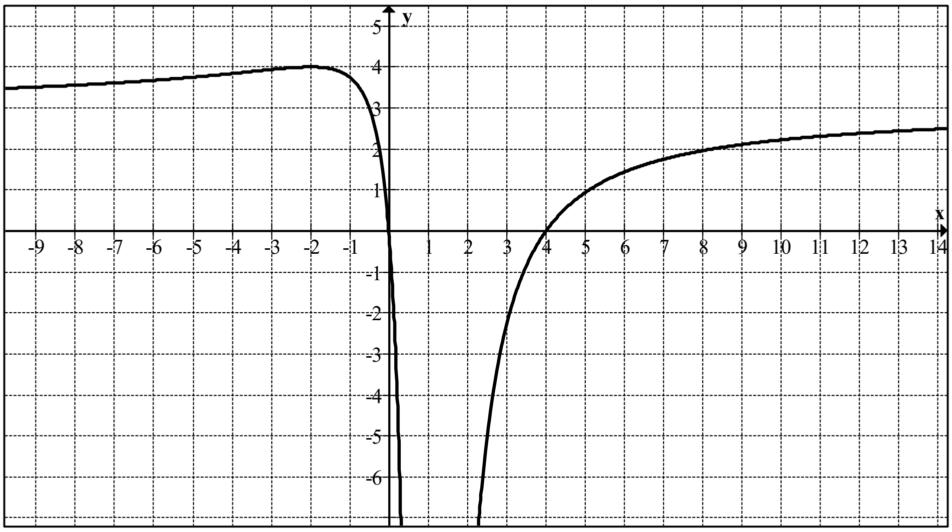
max/min high point H(-2/4)

inflection points inflection point $\underline{I\left(-3.5 / 3.\overline{8}\right)}$

asymptotes vertical: x = 1 (cf. domain)

horizontal: y = 3

graph



b) by differentiation:

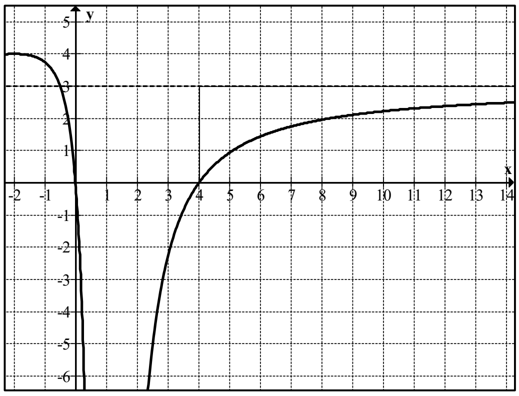
$$\underline{\underline{F'(x)}} = \frac{d}{dx}\left(-6\cdot\ln(x-1)+\frac{9}{x-1}+3x+c\right) = -6\cdot\frac{1}{x-1}-\frac{9}{(x-1)^2}+3$$

$$= \frac{-6\cdot(x-1)-9+3(x-1)^2}{(x-1)^2} = \frac{-6x+6-9+3x^2-6x+3}{(x-1)^2} = \underline{\underline{\frac{3x^2-12x}{(x-1)^2}}}$$

c) $\int\limits_4^\infty (3-f(x))\,dx =$

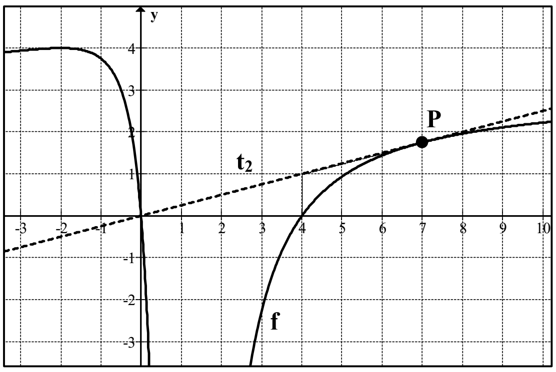
$$\left[3x-\left(-6\cdot\ln(|x-1|)+\frac{9}{x-1}+3x\right)\right]_4^\infty$$

$$= \lim_{u\to\infty}\left[6\cdot\ln(|u-1|)-\frac{9}{u-1}-(6\cdot\ln(3)-3)\right] = \infty$$



d) tangent in point $P\left(7/\frac{7}{4}\right)$: $t(x)=\frac{1}{4}x$,

so t passes through the origin



e) V = 8.55

Exercise 4 [Stochastics]

a) a_1) 60 a_2) 720

b) b_1) 20 b_2) 30

c) P[at least one hit] = 1 – P[no hit] = $1 - 0.35^n \geq 0.999 \rightarrow n \geq 6.6 \rightarrow$ 7 shots

d) d_1) 0.005 d_2) 0.765

e) e_1) 0.096 e_2) 1.64 silver coins